In this chapter, you learn about the use of easily-stated if-then rules to solve problems. In particular, you learn about forward chaining from assertions and backward chaining from hypotheses.

By way of illustration, you learn about two toy systems; one identifies zoo animals, the other bags groceries. These examples are analogous to influential, classic systems that diagnose diseases and configure computers.

You also learn about how to implement rule-based systems. You learn, for example, how search methods can be deployed to determine which of many possible rules are applicable during backward chaining, and you learn how the rete procedure does efficient forward chaining.

When you have finished this chapter, you will understand the key ideas that support many of the useful applications of artificial intelligence. Such applications are often mislabeled expert systems, even though their problem-solving behavior seems more like that of human novices, rather than of human experts.

**RULE-BASED DEDUCTION SYSTEMS**

Rule-based problem-solving systems are built using rules like the following, each of which contains several if patterns and one or more then patterns:
In this section, you learn how rule-based systems work.

Many Rule-Based Systems Are Deduction Systems

A statement that something is true, such as “Stretch has long legs,” or “Stretch is a giraffe,” is an assertion. In all rule-based systems, each if pattern is a pattern that may match one or more of the assertions in a collection of assertions. The collection of assertions is sometimes called working memory.

In many rule-based systems, the then patterns specify new assertions to be placed into working memory, and the rule-based system is said to be a deduction system. In deduction systems, the convention is to refer to each if pattern as an antecedent and to each then pattern as a consequent. Figure 7.1 shows a graphical notation for deduction-oriented antecedent–consequent rules.

Sometimes, however, the then patterns specify actions, rather than assertions—for example, “Put the item into the bag”—in which case the rule-based system is a reaction system.

In both deduction systems and reaction systems, forward chaining is the process of moving from the if patterns to the then patterns, using the if patterns to identify appropriate situations for the deduction of a new assertion or the performance of an action.

†Facts and assertions are subtly different: A fact is something known to be true; an assertion is a statement that something is a fact. Thus, assertions can be false, but facts cannot be false.
During forward chaining, whenever an *if* pattern is observed to match an assertion, the antecedent is **satisfied**. Whenever all the *if* patterns of a rule are satisfied, the rule is **triggered**. Whenever a triggered rule establishes a new assertion or performs an action, it is **fired**.

In deduction systems, all triggered rules generally fire. In reaction systems, however, when more than one rule is triggered at the same time, usually only one of the possible actions is desired, thus creating a need for some sort of conflict-resolution procedure to decide which rule should fire.

**A Toy Deduction System Identifies Animals**

Suppose that Robbie, a robot, wants to spend a day at the zoo. Robbie can perceive basic features, such as color and size, and whether an animal has hair or gives milk, but his ability to identify objects using those features is limited. He can distinguish animals from other objects, but he cannot use the fact that a particular animal has a long neck to conclude that he is looking at a giraffe.

Plainly, Robbie will enjoy the visit more if he can identify the individual animals. Accordingly, Robbie decides to build **ZOOKEEPER**, an identification-oriented deduction system.

Robbie could build **ZOOKEEPER** by creating one *if*-then rule for each kind of animal in the zoo. The consequent side of each rule would be a simple assertion of animal identity, and the antecedent side would be a bulbous enumeration of characteristics sufficiently complete to reject all incorrect identifications.

Robbie decides, however, to build **ZOOKEEPER** by creating rules that produce intermediate assertions. The advantage is that the antecedent–consequent rules involved need have only a few antecedents, making them easier for Robbie to create and use. Using this approach, **ZOOKEEPER** produces chains of conclusions leading to the identification of the animal that Robbie is currently examining.

Now suppose that Robbie’s local zoo contains only seven animals: a cheetah, a tiger, a giraffe, a zebra, an ostrich, a penguin, and an albatross. This assumption simplifies **ZOOKEEPER**, because only a few rules are needed to distinguish one type of animal from another. One such rule, rule **Z1**, determines that a particular animal is a mammal:

\[
\begin{align*}
\text{Z1} & \quad \text{If } \ ?x \ \text{has hair} \\
& \quad \text{then } \ ?x \ \text{is a mammal}
\end{align*}
\]

Note that antecedents and consequents are patterns that contain **variables**, such as \(x\), marked by question-mark prefixes. Whenever a rule is considered, its variables have no values initially, but they acquire values as antecedent patterns are matched to assertions.

Suppose that a particular animal, named Stretch, has hair. Then, if the working memory contains the assertion *Stretch has hair*, the antecedent
pattern, \( ?x \text{ has hair} \), matches that assertion, and the value of \( x \) becomes \textit{Stretch}. By convention, when variables become identified with values, they are said to be \textbf{bound} to those values and the values are sometimes called \textbf{bindings}. Thus, \( x \) is bound to \textit{Stretch} and \textit{Stretch} is \( x \)'s binding.

Once a variable is bound, that variable is replaced by its binding wherever the variable appears in the same or subsequently processed patterns. Whenever the variables in a pattern are replaced by variable bindings, the pattern is said to be \textbf{instantiated}. For example, the consequent pattern, \( ?x \text{ is a mammal} \) becomes \textit{Stretch is a mammal} once instantiated by the variable binding acquired when the antecedent pattern was matched.

Now let us look at other \textsc{zookeeper} rules. Three others also determine biological class:

\begin{itemize}
  \item \textbf{Z2} \hspace{1em} \textbf{If} \hspace{1em} \( ?x \text{ gives milk} \) \hspace{1em} \textbf{then} \hspace{1em} \( ?x \text{ is a mammal} \)
  \item \textbf{Z3} \hspace{1em} \textbf{If} \hspace{1em} \( ?x \text{ has feathers} \) \hspace{1em} \textbf{then} \hspace{1em} \( ?x \text{ is a bird} \)
  \item \textbf{Z4} \hspace{1em} \textbf{If} \hspace{1em} \( ?x \text{ flies} \) \hspace{1em} \( ?x \text{ lays eggs} \) \hspace{1em} \textbf{then} \hspace{1em} \( ?x \text{ is a bird} \)
\end{itemize}

The last of these rules, \textbf{Z4}, has two antecedents. Although it does not really matter for the small collection of animals in \textsc{zookeeper}'s world, some mammals fly and some reptiles lay eggs, but no mammal or reptile does both.

Once \textsc{zookeeper} knows that an animal is a mammal, two rules determine whether that animal is carnivorous. The simpler rule has to do with catching the animal in the act of having its dinner:

\begin{itemize}
  \item \textbf{Z5} \hspace{1em} \textbf{If} \hspace{1em} \( ?x \text{ is a mammal} \) \hspace{1em} \( ?x \text{ eats meat} \) \hspace{1em} \textbf{then} \hspace{1em} \( ?x \text{ is a carnivore} \)
\end{itemize}

If Robbie is not at the zoo at feeding time, various other factors, if available, provide conclusive evidence:

\begin{itemize}
  \item \textbf{Z6} \hspace{1em} \textbf{If} \hspace{1em} \( ?x \text{ is a mammal} \) \hspace{1em} \( ?x \text{ has pointed teeth} \) \hspace{1em} \( ?x \text{ has claws} \) \hspace{1em} \( ?x \text{ has forward-pointing eyes} \) \hspace{1em} \textbf{then} \hspace{1em} \( ?x \text{ is a carnivore} \)
\end{itemize}

All hooved animals are ungulates:
Z7 If $x$ is a mammal
    $x$ has hoofs
then $x$ is an ungulate

If Robbie has a hard time looking at the feet, ZOOKEEPER may still have a chance because all animals that chew cud are also ungulates:

Z8 If $x$ is a mammal
    $x$ chews cud
then $x$ is an ungulate

Now that Robbie has rules that divide mammals into carnivores and ungulates, it is time to add rules that identify specific animal identities. For carnivores, there are two possibilities:

Z9 If $x$ is a carnivore
    $x$ has tawny color
    $x$ has dark spots
then $x$ is a cheetah

Z10 If $x$ is a carnivore
    $x$ has tawny color
    $x$ has black strips
then $x$ is a tiger

Strictly speaking, the basic color is not useful because both of the carnivores are tawny. However, there is no need for information in rules to be minimal. Moreover, antecedents that are superfluous now may become essential later as new rules are added to deal with other animals.

For the ungulates, other rules separate the total group into two possibilities:

Z11 If $x$ is an ungulate
    $x$ has long legs
    $x$ has long neck
    $x$ has tawny color
    $x$ has dark spots
then $x$ is a giraffe

Z12 If $x$ is an ungulate
    $x$ has white color
    $x$ has black stripes
then $x$ is a zebra

Three more rules are needed to handle the birds:
Z13 If  
   ?x is a bird 
   ?x does not fly 
   ?x has long legs 
   ?x has long neck 
   ?x is black and white 
then  ?x is an ostrich

Z14 If  
   ?x is a bird 
   ?x does not fly 
   ?x swims 
   ?x is black and white 
then  ?x is a penguin

Z15 If  
   ?x is a bird 
   ?x is a good flyer 
then  ?x is an albatross

Now that you have seen all the rules in ZOOKEEPER, note that the animals evidently share many features. Zebras and tigers have black stripes; tigers, cheetahs, and giraffes have a tawny color; giraffes and ostriches have long legs and a long neck; and ostriches and penguins are black and white.

To learn about how forward chaining works, suppose that Robbie is at the zoo and is about to analyze an unknown animal, Stretch, using ZOOKEEPER. Further suppose that the following six assertions are in working memory:

Stretch has hair.  
Stretch chews cud.  
Stretch has long legs.  
Stretch has a long neck.  
Stretch has tawny color.  
Stretch has dark spots.

Because Stretch has hair, rule Z1 fires, establishing that Stretch is a mammal. Because Stretch is a mammal and chews cud, rule Z8 establishes that Stretch is an ungulate.

At this point, all the antecedents for rule Z11 are satisfied. Evidently, Stretch is a giraffe.

**Rule-Based Systems Use a Working Memory and a Rule Base**

As you have seen in the ZOOKEEPER system, one of the key representations in a rule-based system is the working memory:
A **working memory** is a representation
In which

- Lexically, there are application-specific symbols and pattern symbols.
- Structurally, assertions are lists of application-specific symbols, and patterns are lists of application-specific symbols and pattern symbols.
- Semantically, the assertions denote facts in some world.

With constructors that

- Add an assertion to working memory

With readers that

- Produce a list of the matching assertions in working memory, given a pattern

Another key representation is the rule base:

A **rule base** is a representation
In which

- Lexically, there are application-specific symbols and pattern symbols.
- Structurally, patterns are lists of application-specific symbols and pattern symbols, and rules consist of patterns. Some of these patterns constitute the rule’s *if* patterns; the others constitute the rule’s *then* pattern.
- Semantically, rules denote constraints that enable procedures to seek new assertions or to validate a hypothesis.

With constructors that

- Construct a rule, given an ordered list of *if* patterns and a *then* pattern

With readers that

- Produce a list of a given rule’s *if* patterns
- Produce a list of a given rule’s *then* patterns

Thus, **ZOOKEEPER** uses instances of these representations that are specialized to animal identification. **ZOOKEEPER** itself can be expressed in procedural English, as follows:
To identify an animal with ZOOKEEPER (forward-chaining version),

▷ Until no rule produces a new assertion or the animal is identified,

▷ For each rule,
   ▷ Try to support each of the rule’s antecedents by matching it to known facts.
   ▷ If all the rule’s antecedents are supported, assert the consequent unless there is an identical assertion already.
   ▷ Repeat for all matching and instantiation alternatives.

Thus, assertions flow through a series of antecedent–consequent rules from given assertions to conclusions, as shown in the history recorded in figure 7.2. In such diagrams, sometimes called inference nets, the O-shaped objects represent rules, whereas vertical bars denote given assertions and vertical boxes denote deduced assertions.

**Deduction Systems May Run Either Forward or Backward**

So far, you have learned about a deduction-oriented rule-based system that works from given assertions to new, deduced assertions. Running this
way, a system exhibits forward chaining. Backward chaining is also possible: A rule-based system can form a hypothesis and use the antecedent–consequent rules to work backward toward hypothesis-supporting assertions.

For example, ZOOKEEPER might form the hypothesis that a given animal, Swifty, is a cheetah and then reason about whether that hypothesis is viable. Here is a scenario showing how things work out according to such a backward-chaining approach:

- ZOOKEEPER forms the hypothesis that Swifty is a cheetah. To verify the hypothesis, ZOOKEEPER considers rule Z9, which requires that Swifty is a carnivore, that Swifty has a tawny color, and that Swifty has dark spots.

- ZOOKEEPER must check whether Swifty is a carnivore. Two rules may do the job, namely rule Z5 and rule Z6. Assume that ZOOKEEPER tries rule Z5 first.

- ZOOKEEPER must check whether Swifty is a mammal. Again, there are two possibilities, rule Z1 and rule Z2. Assume that ZOOKEEPER tries rule Z1 first. According to that rule, Swifty is a mammal if Swifty has hair.

- ZOOKEEPER must check whether Swifty has hair. Assume ZOOKEEPER already knows that Swifty has hair. So Swifty must be a mammal, and ZOOKEEPER can go back to working on rule Z5.

- ZOOKEEPER must check whether Swifty eats meat. Assume ZOOKEEPER cannot tell at the moment. ZOOKEEPER therefore must abandon rule Z5 and try to use rule Z6 to establish that Swifty is a carnivore.

- ZOOKEEPER must check whether Swifty is a mammal. Swifty is a mammal, because this was already established when trying to satisfy the antecedents in rule Z5.

- ZOOKEEPER must check whether Swifty has pointed teeth, has claws, and has forward-pointing eyes. Assume ZOOKEEPER knows that Swifty has all these features. Evidently, Swifty is a carnivore, so ZOOKEEPER can return to rule Z9, which started everything done so far.

- Now ZOOKEEPER must check whether Swifty has a tawny color and dark spots. Assume ZOOKEEPER knows that Swifty has both features. Rule Z9 thus supports the original hypothesis that Swifty is a cheetah, and ZOOKEEPER therefore concludes that Swifty is a cheetah.

Thus, ZOOKEEPER is able to work backward through the antecedent–consequent rules, using desired conclusions to decide for what assertions it should look. A backward-moving chain develops, as dictated by the following procedure:
To identify an animal with ZOOKEEPER (backward-chaining version),

- Until all hypotheses have been tried and none have been supported or until the animal is identified,
- For each hypothesis,
  - For each rule whose consequent matches the current hypothesis,
    - Try to support each of the rule's antecedents by matching it to assertions in working memory or by backward chaining through another rule, creating new hypotheses. Be sure to check all matching and instantiation alternatives.
    - If all the rule's antecedents are supported, announce success and conclude that the hypothesis is true.

In the example, backward chaining ends successfully, verifying the hypothesis, as shown in figure 7.3. The chaining ends unsuccessfully if any required antecedent assertions cannot be supported.
The Problem Determines Whether Chaining Should Be Forward or Backward

Many deduction-oriented antecedent–consequent rule systems can chain either forward or backward, but which direction is better? This subsection describes several rules of thumb that may help you to decide.

Most important, you want to think about how the rules relate facts to conclusions. Whenever the rules are such that a typical set of facts can lead to many conclusions, your rule system exhibits a high degree of fan out, and a high degree of fan out argues for backward chaining. On the other hand, whenever the rules are such that a typical hypothesis can lead to many questions, your rule system exhibits a high degree of fan in, and a high degree of fan in argues for forward chaining.

- If the facts that you have or may establish can lead to a large number of conclusions, but the number of ways to reach the particular conclusion in which you are interested is small, then there is more fan out than fan in, and you should use backward chaining.
- If the number of ways to reach the particular conclusion in which you are interested is large, but the number of conclusions that you are likely to reach using the facts is small, then there is more fan in than fan out, and you should use forward chaining.

Of course, in many situations, neither fan out nor fan in dominates, leading you to other considerations:

- If you have not yet gathered any facts, and you are interested in only whether one of many possible conclusions is true, use backward chaining.

Suppose, for example, that you do not care about the identity of an animal. All you care about is whether it is a carnivore. By backward chaining from the carnivore hypothesis, you ensure that all the facts you gather are properly focused. You may ask about the animal's teeth, but you will never ask about the animal's color.

- If you already have in hand all the facts you are ever going to get, and you want to know everything you can conclude from those facts, use forward chaining.

Suppose, for example, that you have had a fleeting glimpse of an animal that has subsequently disappeared. You want to know what you can deduce about the animal. If you were to backward chain, you would waste time pursuing hypotheses that lead back to questions you can no longer answer because the animal has disappeared. Accordingly, you are better off if you forward chain.

**RULE-BASED REACTION SYSTEMS**

In deduction systems, the if parts of some if–then rules specify combinations of assertions, and the then part specifies a new assertion to be deduced
**Mycin Diagnoses Bacterial Infections of the Blood**

ZOOKEEPER is based on MYCIN, a well-known rule-based deduction system. Historically, the MYCIN system played a major role in stimulating research interest in rule-based expert systems.

MYCIN's expertise lies in the domain of bacterial infections. Physicians usually must begin antibiotic treatment for patients who have bacterial infections without knowing exactly which organism is the culprit. There is no time to wait for definitive laboratory culture evidence, which accumulates too slowly. For the desperately sick, therapy must begin at once—not 2 days from at once. This requirement leaves two choices: the physician can either prescribe a broad-spectrum drug that covers all possibilities, or she can prescribe a better, disease-specific drug.

MYCIN helps the physician to prescribe disease-specific drugs. MYCIN informs itself about particular cases by requesting information from the physician about a patient's symptoms, general condition, history, and laboratory-test results that can be obtained easily and quickly. At each point, the question MYCIN asks is determined by MYCIN's current hypothesis and the answers to all previous questions. Thus, the questions start as though taken from a checklist, but the questions then vary as evidence builds. Here is an excerpt from a sample session:

> What is the patient's name?
  
  John Doe.

> Male or female?
  
  Male.

> Age?
  
  He is 55.

> Have you obtained positive cultures indicating general type?
  
  Yes.

> What type of infection is it?
  
  Primary bacteremia.

> When did symptoms first appear?
  
  May 5.

> Let's call the most recent positive culture C1. From what site was C1 taken?
  
  From the blood.

> When?
  
  May 9.

> Let's call the first significant organism from this culture U1. Do you know the identity of U1?
  
  No.

> Is U1 a rod or a coccus or something else?
  
  Rod.

> What is the gram stain of U1?
  
  Gram-negative.

> Have there been positive cultures from sites that are not associated with the infections about which you are seeking advice?
  
  No.

> Did U1 grow aerobically?
  
  Yes.
When it is satisfied that it has a reasonably good grasp of the situation, MYCIN announces its diagnosis:

> My recommendation will be based on the opinion that the identity of U1 may be
  1. Pseudomonas-aeruginosa
  2. Klebsiella-pneumoniae
  3. E. coli
  4. Bacteroides-fragilis
  5. Enterobacter
  6. Proteus-nonmirabilis.

After asking a few more questions about the patient, MYCIN announces its recommendation:

> On a scale of 0 to 4, where higher numbers indicate greater severity, how would you rate the patient's degree of sickness?
  3.
> Does the patient have a clinically significant allergic reaction to any antimicrobial agent?
  No.
> How much does the patient weigh?
  70 kilograms.
> My first recommendation is as follows: to cover for items 1, 2, 3, 5, and 6, give gentamycin using a dose of 119 mg (1.7 mg/kg) q8h IV [or IM] for 10 days. Modify dose in renal failure. Also, to cover for item 4, give clindamycin using a dose of 505 mg (8.5 mg/kg) q8h IV [or IM] for 14 days. If diarrhea or other gastrointestinal symptoms develop, check for pseudomembranous colitis.

MYCIN's pool of knowledge consists of approximately 500 antecedent–consequent rules, which give MYCIN the ability to recognize about 100 causes of bacterial infections. The following rule is typical:

M88  If  ?x's type is primary bacteremia  
      the suspected entry point of ?x is the gastrointestinal tract  
      the site of the culture of ?x is one of the sterile sites  
      then  there is evidence that ?x is bacteroides

MYCIN is a backward-chaining system, because physicians prefer to think about one hypothesis at a time. By sticking with the questions that are relevant to a particular hypothetical conclusion, the questioning is guaranteed to remain relevant to that hypothesis. A forward-running system can jump around, working first toward one conclusion and then toward another, seemingly at random.

Another reason why MYCIN was designed to be a backward-chaining system is that backward chaining simplifies the creation of an English-language interface. The interface needs to deal only with answers to specific questions, rather than with free-form, imaginative text.
directly from the triggering combination. In reaction systems, which are introduced in this section, the if parts specify the conditions that have to be satisfied and the then part specifies an action to be undertaken. Sometimes, the action is to add a new assertion; sometimes it is to delete an existing assertion; sometimes, it is to execute some procedure that does not involve assertions at all.

**A Toy Reaction System Bags Groceries**

Suppose that Robbie has just been hired to bag groceries in a grocery store. Because he knows little about bagging groceries, he approaches his new job by creating BAGGER, a rule-based reaction system that decides where each item should go.

After a little study, Robbie decides that BAGGER should be designed to take four steps:

1. The check-order step: BAGGER analyzes what the customer has selected, looking over the groceries to see whether any items are missing, with a view toward suggesting additions to the customer.
2. The bag-large-items step: BAGGER bags the large items, taking care to put the big bottles in first.
3. The bag-medium-items step: BAGGER bags the medium items, taking care to put frozen ones in freezer bags.
4. The bag-small-items step: BAGGER bags the small items.

Now let us see how this knowledge can be captured in a rule-based reaction system. First, BAGGER needs a working memory. The working memory must contain assertions that capture information about the items to be bagged. Suppose that those items are the items listed in the following table:

<table>
<thead>
<tr>
<th>Item</th>
<th>Container type</th>
<th>Size</th>
<th>Frozen?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread</td>
<td>plastic bag</td>
<td>medium</td>
<td>no</td>
</tr>
<tr>
<td>Glop</td>
<td>jar</td>
<td>small</td>
<td>no</td>
</tr>
<tr>
<td>Granola</td>
<td>cardboard box</td>
<td>large</td>
<td>no</td>
</tr>
<tr>
<td>Ice cream</td>
<td>cardboard carton</td>
<td>medium</td>
<td>yes</td>
</tr>
<tr>
<td>Potato chips</td>
<td>plastic bag</td>
<td>medium</td>
<td>no</td>
</tr>
<tr>
<td>Pepsi</td>
<td>bottle</td>
<td>large</td>
<td>no</td>
</tr>
</tbody>
</table>

Next, BAGGER needs to know which step is the current step, which bag is the current bag, and which items already have been placed in bags. In the following example, the first assertion identifies the current step as the check-order step, the second identifies the bag as Bag1, and the remainder indicate what items are yet to be bagged:
Step is check-order
Bag1 is a bag
Bread is to be bagged
Glop is to be bagged
Granola is to be bagged
Ice cream is to be bagged
Potato chips are to be bagged

Note that working memory contains an assertion that identifies the step. Each of the rules in BAGGER's rule base tests the step name. Rule B1, for example, is triggered only when the step is the check-order step:

B1   If     step is check-order
     then  potato chips are to be bagged
     there is no Pepsi to be bagged
     ask the customer whether he would like a bottle of Pepsi

The purpose of rule B1 is to be sure the customer has something to drink to go along with potato chips, because potato chips are dry and salty. Note that rule B1's final condition checks that a particular pattern does not match any assertion in working memory.

Now let us move on to a rule that moves BAGGER from the check-order step to the bag-large-items step:

B2   If     step is check-order
     then  step is no longer check-order
     step is bag-large-items

Note that the first of rule B2's actions deletes an assertion from working memory. Deduction systems are assumed to deal with static worlds in which nothing that is shown to be true can ever become false. Reaction systems, however, are allowed more freedom. Sometimes, that extra freedom is reflected in the rule syntax through the break up of the action part of the rule, marked by then, into two constituent parts, marked by delete and add. When you use this alternate syntax, rule B2 looks like this:

B2 (add–delete form)
If     step is check-order
delete  step is check-order
add     step is bag-large-items

The remainder of BAGGER's rules are expressed in this more transparent add–delete syntax.

At first, rule B2 may seem dangerous, for it looks as though it could prevent rule B1 from doing its legitimate and necessary work. There is no problem, however. Whenever you are working with a reaction system, you adopt a suitable conflict-resolution procedure to determine which rule
to fire among many that may be triggered. BAGGER uses the simplest conflict-resolution strategy, rule ordering, which means that the rules are arranged in a list, and the first rule triggered is the one that is allowed to fire. By placing rule B2 after rule B1, you ensure that rule B1 does its job before rule B2 changes the step to bag-large-items. Thus, rule B2 changes the step only when nothing else can be done.

Use of the rule-ordering conflict resolution helps you out in other ways as well. Consider, for example, the first two rules for bagging large items:

B3  If       step is bag-large-items
          a large item is to be bagged
          the large item is a bottle
          the current bag contains < 6 large items
    delete  the large item is to be bagged
    add     the large item is in the current bag

B4  If       step is bag-large-items
          a large item is to be bagged
          the current bag contains < 6 large items
    delete  the large item is to be bagged
    add     the large item is in the current bag

Big items go into bags that do not have too many items already, but the bottles—being heavy—go in first. The placement of rule B3 before rule B4 ensures this ordering.

Note that rules B3 and B4 contain a condition that requires counting, so BAGGER must do more than assertion matching when looking for triggered rules. Most rule-based systems focus on assertion matching, but provide an escape hatch to a general-purpose programming language when you need to do more than just match an antecedent pattern to assertions in working memory.

Evidently, BAGGER is to add large items only when the current bag contains fewer than six items.† When the current bag contains six or more items, BAGGER uses rule B5 to change bags:

B5  If       step is bag-large-items
          a large item is to be bagged
          an empty bag is available
    delete  the current bag is the current bag
    add     the empty bag is the current bag

Finally, another step-changing rule moves BAGGER to the next step:

†Perhaps a better BAGGER system would use volume to determine when bags are full; to deal with volume, however, would require general-purpose computation that would make the example unnecessarily complicated, albeit more realistic.
B6  If step is bag-large-items
    delete step is bag-large-items
    add step is bag-medium-items

Let us simulate the result of using these rules on the given database. As we start, the step is check-order. The order to be checked contains potato chips, but no Pepsi. Accordingly, rule B1 fires, suggesting to the customer that perhaps a bottle of Pepsi would be nice. Let us assume that the customer goes along with the suggestion and fetches a bottle of Pepsi.

Inasmuch as there are no more check-order rules that can fire, other than rule B2, the one that changes the step to bag-large-items, the step becomes bag-large-items.

Now, because the Pepsi is a large item in a bottle, the conditions for rule B3 are satisfied, so rule B3 puts the Pepsi in the current bag. Once the Pepsi is in the current bag, the only other large item is the box of granola, which satisfies the conditions of rule B4, so it is bagged as well, leaving the working memory in the following condition:

Step is bag-medium-items
Bag1 contains Pepsi
Bag1 contains granola
Bread is to be bagged
Glop is to be bagged
Ice cream is to be bagged
Potato chips are to be bagged

Now it is time to look at rules for bagging medium items.

B7  If step is bag-medium-items
    a medium item is frozen, but not in a freezer bag
    delete the medium item is not in a freezer bag
    add the medium item is in a freezer bag

B8  If step is bag-medium-items
    a medium item is to be bagged
    the current bag is empty or contains only medium items
    the current bag contains no large items
    delete the current bag contains < 12 medium items
    add the medium item is to be bagged
    the medium item is in the current bag

B9  If step is bag-medium-items
    a medium item is to be bagged
    an empty bag is available
    delete the current bag is the current bag
    add the empty bag is the current bag
Note that the fourth condition that appears in rule B8 prevents BAGGER from putting medium items in a bag that already contains a large item. If there is a bag that contains a large item, rule B9 starts a new bag.

Also note that rule B7 and rule B8 make use of the rule-ordering conflict-resolution procedure. If both rule B7 and rule B8 are triggered, rule B7 is the one that fires, ensuring that frozen things are placed in freezer bags before bagging.

Finally, when there are no more medium items to be bagged, neither rule B7 nor rule B8 is triggered; instead, rule B10 is triggered and fires, changing the step to bag-small-items:

B10  If step is bag-medium-items
     delete step is bag-medium-items
     add   step is bag-small-items

At this point, after execution of all appropriate bag-medium-item rules, the situation is as follows:

Step is bag-small-items
Bag1 contains Pepsi
Bag1 contains granola
Bag2 contains bread
Bag2 contains ice cream (in freezer bag)
Bag2 contains potato chips
Glop is to be bagged

Note that, according to simple rules used by BAGGER, medium items do not go into bags with large items. Similarly, conditions in rule B11 ensure that small items go in their own bag:

B11  If step is bag-small-items
     a small item is to be bagged
     the current bag contains no large items
     the current bag contains no medium items
     the bag contains < 18 small items
     delete the small item is to be bagged
     add   the small item is in the current bag

BAGGER needs a rule that starts a new bag:

B12  If step is bag-small-items
     a small item is to be bagged
     an empty bag is available
     delete the current bag is the current bag
     add   the empty bag is the current bag

Finally, BAGGER needs a rule that detects when bagging is complete:
B13  If step is bag-small-items
    delete step is bag-small-items
    add step is done

After all rules have been used, everything is bagged:

Step is done
Bag1 contains Pepsi
Bag1 contains granola
Bag2 contains bread
Bag2 contains ice cream (in freezer bag)
Bag2 contains potato chips
Bag3 contains glop

**Reaction Systems Require Conflict Resolution Strategies**

Forward-chaining deduction systems do not need strategies for conflict resolution because every rule presumably produces reasonable assertions, so there is no harm in firing all triggered rules. But in reaction systems, when more than one rule is triggered, you generally want to perform only one of the possible actions, thus requiring a *conflict-resolution strategy* to decide which rule actually fires. So far, you have learned about rule ordering:

- *Rule ordering.* Arrange all rules in one long prioritized list. Use the triggered rule that has the highest priority. Ignore the others.

Here are other possibilities:

- *Context limiting.* Reduce the likelihood of conflict by separating the rules into groups, only some of which are active at any time.
- *Specificity ordering.* Whenever the conditions of one triggered rule are a superset of the conditions of another triggered rule, use the superset rule on the ground that it deals with more specific situations.
- *Data ordering.* Arrange all possible assertions in one long prioritized list. Use the triggered rule that has the condition pattern that matches the highest priority assertion in the list.
- *Size ordering.* Use the triggered rule with the toughest requirements, where *toughest* means the longest list of conditions.
- *Recency ordering.* Use the least recently used rule.

Of course, the proper choice of a conflict resolution strategy for a reaction system depends on the situation, making it difficult or impossible to rely on a fixed conflict resolution strategy or combination of strategies. An alternative is to think about which rule to fire as another problem to be solved. An elegant example of such problem solving is described in Chapter 8 in the introduction of the SOAR problem solving architecture.
PROCEDURES FOR FORWARD AND BACKWARD CHAINING

In this section, you learn more about rule-based systems. The focus is on how to do forward and backward chaining using well-known methods for exploring alternative variable bindings.

Depth-First Search Can Supply Compatible Bindings for Forward Chaining

One simple way to do forward chaining is to cycle through the rules, looking for those that lead to new assertions once the consequents are instantiated with appropriate variable bindings:

To forward chain (coarse version),

▷ Until no rule produces a new assertion,
  ▷ For each rule,
    ▷ For each set of possible variable bindings determined by matching the antecedents to working memory,
    ▷ Instantiate the consequent.
    ▷ Determine whether the instantiated consequent is already asserted. If it is not, assert it.

For an example, let us turn from the zoo to the track, assuming the following assertions are in working memory:

Comet is-a horse
Prancer is-a horse
Comet is-a-parent-of Dasher
Comet is-a-parent-of Prancer
Prancer is fast
Dasher is-a-parent-of Thunder
Thunder is fast
Thunder is-a horse
Dasher is-a horse

Next, let us agree that a horse who is the parent of something fast is valuable. Translating this knowledge into an if-then rule produces the following:

Parent Rule
If
 ?x is-a horse
 ?x is-a-parent-of ?y
 ?y is fast
then
 ?x is valuable
XCON Configures Computer Systems

BAGGER is based on XCON, a well-known rule-based deduction system. Historically, the XCON system played a major role in stimulating commercial interest in rule-based expert systems.

XCON's domain is computer-system components. When a company buys a big mainframe computer, it buys a central processor, memory, terminals, disk drives, tape drives, various peripheral controllers, and other paraphernalia. All these components must be arranged sensibly along input–output busses. Moreover, all the electronic modules must be placed in the proper kind of cabinet in a suitable slot of a suitable backplane.

Arranging all the components is a task called configuration. Doing configuration can be tedious, because a computer-component family may have hundreds of possible options that can be organized in an unthinkable number of combinations.

To do configuration, XCON uses rules such as the following:

X1 If the context is doing layout and assigning a power supply an sbi module of any type has been put in a cabinet the position that the sbi module occupies is known there is space available for a power supply there is no available power supply then add an appropriate power supply

X2 If the context is doing layout and assigning a power supply an sbi module of any type has been put in a cabinet the position the sbi module occupies is known there is space available for a power supply there is an available power supply then put the power supply in the cabinet in the available space

The first rule, X1, acts rather like the one in BAGGER that asks the customer whether he wants a bottle of Pepsi if the order contains potato chips but no beverage. The second rule, X2, is a typical insertion rule. The context mentioned in both rules is a combination of the top-level step and a substep. The context is changed by rules such as the following:

X3 If the current context is x then deactivate the x context activate the y context

Rule X3 has the effect of deleting one item from the context designation and adding another. It fires only if no other rule associated with the context triggers.

XCON has nearly 10,000 rules and knows the properties of several hundred component types for VAX computers, made by Digital Equipment Corporation. XCON routinely handles orders involving 100 to 200 components. It is representative of many similar systems for marketing and manufacturing.
Figure 7.4 During forward chaining, binding commitments can be arranged in a tree, suggesting that ordinary search methods can be used to find one or all of the possible binding sets. Here, the parent rule’s first antecedent leads to four possible bindings for \( x \), and the rule’s second antecedent, given that \( x \) is bound to Comet, leads to two possible bindings for \( y \).

Now, if there is a binding for \( x \) and a binding for \( y \) such that each antecedent corresponds to an assertion when the variables are replaced by their bindings, then the rule justifies the conclusion that the thing bound to \( x \) is valuable. For example, if \( x \) is bound to Comet and \( y \) is bound to Prancer, then each of the antecedents corresponds to an assertion—namely Comet is-a horse, Comet is-a-parent-of Prancer, and Prancer is fast. Accordingly, Comet must be valuable.

To conduct a search for binding pairs, you can start by matching the first antecedent against each assertion. As shown in figure 7.4, there are four matches and four corresponding binding choices for \( x \) in the antecedent ?\( x \) is-a horse, because Comet, Prancer, Thunder, and Dasher are all horses.

Next, proceeding in the depth-first search style, assume that \( x \)'s binding should be Comet, which is the binding produced by the first match. Then, with \( x \) bound to Comet, the second assertion, after instantiation, is Comet is-a-parent-of ?\( y \). Matching this second instantiated antecedent against each assertion produces two matches, because Comet is a parent of both Dasher and Prancer. Thus, there are two binding choices for \( y \) given that \( x \) is bound to Comet.

Figure 7.4 show how the \( x \) and \( y \) choices fit together. Evidently, each of the two antecedents examined so far produces binding choices that can be arranged in a search tree.

Traveling along the leftmost branch, with \( x \) bound to Comet and \( y \) bound to Dasher, you proceed to the third antecedent, which becomes Dasher is fast when instantiated with \( y \)'s binding. This instantiated an-
Figure 7.5 Two paths extend from the root down through the levels corresponding to three rule antecedents. Evidently, there are two binding sets that satisfy the rule.

If an antecedent fails to match any assertion, however, so you have to look farther for an acceptable combination. You do not have to look far, because the combination with $x$ bound to Comet, as before, and $y$ bound to Prancer leads to the instantiation of the third antecedent as *Prancer is fast*, which does match an assertion. Accordingly, you can conclude that the combination with $x$ bound to Comet and $y$ bound to Prancer is a combination that jumps over all the antecedent hurdles. You can use this combination to instantiate the consequent, producing *Comet is valuable*.

As shown in figure 7.4, there are three other choices for $x$ bindings. Among these, if $x$ is bound to Prancer or Thunder, then the second assertion, once instantiated, becomes *Prancer is-a-parent-of ?y* or *Thunder is-a-parent-of ?y*, both of which fail to match any assertion. If Dasher is the proper binding, then *Dasher is-a-parent-of ?y* matches just one assertion, *Dasher is-a-parent-of Thunder*, leaving only Thunder as a choice for $y$'s binding. With $x$ bound to Dasher and $y$ bound to Thunder, the third instantiated antecedent is *Thunder is fast*, which matches an assertion, leading to the conclusion, as shown in figure 7.5, that the Dasher-Thunder combination also jumps over all the hurdles, suggesting that Dasher is valuable too.

From this example, several points of interest emerge. First, you can see that each path in the search tree corresponds to a set of binding commitments. Second, each antecedent matches zero or more assertions given the bindings already accumulated along a path, and each successful match produces a branch. Third, the depth of the search tree is always equal
to the number of antecedents. Fourth, you have a choice, as usual, about how you search the tree. Exhaustive, depth-first, left-to-right search is the usual method when the objective is to find all possible ways that a rule can be deployed. This method is the one exhibited in the following procedure:

---

To forward chain (detailed version),
- Until no rule produces a new assertion,
  - For each rule,
    - Try to match the first antecedent with an existing assertion. Create a new binding set with variable bindings established by the match.
    - Using the existing variable bindings, try to match the next antecedent with an existing assertion. If any new variables appear in this antecedent, augment the existing variable bindings.
    - Repeat the previous step for each antecedent, accumulating variable bindings as you go, until,
      - There is no match with any existing assertion using the binding set established so far. In this case, back up to a previous match of an antecedent to an assertion, looking for an alternative match that produces an alternative, workable binding set.
      - There are no more antecedents to be matched. In this case,
        - Use the binding set in hand to instantiate the consequent.
        - Determine if the instantiated consequent is already asserted. If not, assert it.
        - Back up to the most recent match with unexplored bindings, looking for an alternative match that produces a workable binding set.
        - There are no more alternatives matches to be explored at any level.

---

**Depth-First Search Can Supply Compatible Bindings for Backward Chaining**

You learned that forward chaining can be viewed as searching for variable-binding sets such that, for each set, all antecedents correspond to assertions once their variables are replaced by bindings from the set.

Backward chaining can be treated in the same general way, but there are a few important differences and complications. In particular, you start
by matching a hypothesis both against existing assertions and against rule consequents.

Suppose, for example, that you are still working with horses using the same rules and assertions in working memory as before. Next, suppose that you want to show that Comet is valuable; in other words, suppose that you want to verify the hypothesis, *Comet is valuable*. You fail to find a match for *Comet is valuable* among the assertions, but you succeed in matching the hypothesis with the rule consequent, $?x$ is valuable. The success leads you to attempt to match the antecedents, presuming that $x$ is bound to Comet.

Happily, the instantiated first antecedent, *Comet is-a horse*, matches an assertion, enabling a search for the instantiated second antecedent, *Comet is-a-parent-of y*. This second antecedent leads to two matches, one with the assertion *Comet is-a-parent-of Dasher* and one with the assertion *Comet is-a-parent-of Prancer*. Accordingly, the search branches, as shown in figure 7.6.

Along the left branch, $y$ is bound to Dasher, leading to a futile attempt to match the third antecedent *Dasher is fast* to an assertion. Along the right branch, however, $y$ is bound to Prancer, leading to a successful attempt to match *Prancer is fast* to an assertion.

Evidently, the hypothesis, *Comet is valuable*, is supported by the combination of the given rule and the given assertions because a binding set, discovered by search, connects the hypothesis with the assertions via the rule.

The search is more complicated, however, when the hypothesis itself contains a variable. Suppose that the question is “Who is valuable?” rather than “Is Comet valuable?” Then, the hypothesis itself, $?z$ is valuable, contains a variable, $z$.

This new hypothesis, like the hypothesis *Comet is valuable*, matches no assertions but does match the consequent, $?x$ is valuable. Now, however, you have a match between two variables, $z$ and $x$, instead of a constant, Comet, and a variable, $x$.

Accordingly, now that it is time to match the first antecedent with the assertions, you go into the match with $z$ bound to $x$. The variable $x$ is not bound to anything, however, so the match of the first antecedent proceeds unfettered, as though the chaining were forward. There are four possible matches of the first antecedent to assertions, with $x$ bound to any one of Comet, Prancer, Thunder, or Dasher. Then, assuming $x$'s binding should be Comet, and working through the bindings allowed by the next two assertions, you are led to one of the results shown in figure 7.7, with $z$ bound to $x$, $x$ bound to Comet, and $y$ bound to Prancer.

The fact that $z$, the variable in the hypothesis, matches $x$, a variable in a rule, need cause you no serious pause. The only additional task you need to perform is to instantiate all the way to constants whenever you have an
option to continue instantiating. Thus, you first replace \( z \) by \( x \), and then you replace \( z \) by Comet, producing an instantiated hypothesis of *Comet is valuable*.

At this point, you could, if you wished, continue to look for other ways to bind variables so as to find other valuable horses.

The search is still more complicated when more than one rule can provide a variable binding. Suppose, for example, that you have the hypothesis with the variable, \( ?z \) is *valuable*, but that you now add a new rule and two new assertions:

**Winner Rule**

If \(?w\) is-a winner

then \(?w\) is fast
During backward chaining, hypothesis variables can be bound not only to assertion constants but also to consequent variables. Here, the hypothesis variable z is bound to the consequent variable x. Ultimately, two binding sets are found, establishing that two horses are valuable.

Dasher  Is-a  Winner
Prancer  Is-a  Winner

Now, the search proceeds as before, as shown in figure 7.8, until it is time to find a match for the third antecedent, \( ?y \text{ is fast} \). The first time, with y bound to Dasher, there is no matching assertion, but there is a match between the second rule's consequent w is fast and the instantiated first-rule antecedent, Dasher is fast. Consequently, w becomes bound to Dasher, and an effort is made to find a match between the second rule's instantiated antecedent, Dasher is-a winner, against an assertion. Because there is a match with one of the two new assertions, you can conclude that Dasher is indeed fast, which means that the original hypothesis, ?z is valuable, can be connected via rules to assertions using the binding set with w and y bound to Dasher, x bound to Comet, and z bound to x. To instantiate the hypothesis with this binding set, you first replace z with x, and then replace x with Comet.

Note that you can gain nothing by trying to find a second match for the instantiated antecedent, Comet is-a-parent-of y, because the ultimate conclusion, that Comet is valuable, has already been reached. Nevertheless,
most binding-set programs are not smart enough to realize that nothing is to be gained, so they look for other ways to bind y using Comet is-a-parent-of y.

As shown in figure 7.8, y can be bound to Prancer, which leads to an attempt to match the third antecedent, Prancer is fast with an assertion; the match succeeds, reaffirming, with a different binding set, that Comet is valuable.

Displaying even more energy, most binding programs not only note that the instantiated antecedent, Prancer is fast, is in the database; they also note, as shown in figure 7.8, that there is a rule that links Prancer is fast to the assertion Prancer is-a winner, thus reaffirming, for a third time, with a binding set that includes a binding for w, that Comet is valuable.
Similarly, when searching for evidence that Dasher is valuable, the instantiated antecedent, \textit{Thunder is fast}, is not only matched to an assertion, it is matched to the consequent, \textit{w is fast}, in the second rule. This time, however, the instantiated antecedent in the second rule, \textit{w is\text{-}a\ winner}, does not match any assertion, so there remains just one way of showing that Dasher is valuable.

In summary, the backward-chaining procedure moves from the initial hypothesis, through rules, to known facts, establishing variable bindings in the process. When the initial hypothesis matches the consequent of a rule, you create a binding set. Additional bindings are added to the initial binding set as the backward-chaining procedure works on the antecedents, and still more bindings are added when the procedure chains through an antecedent to the consequent of another rule. The following procedure summarizes:

To backward chain,

\begin{itemize}
  \item Find a rule whose consequent matches the hypothesis (or antecedent) and create a binding set (or augment the existing binding set).
  \item Using the existing binding set, look for a way to deal with the first antecedent,
    \begin{itemize}
      \item Try to match the antecedent with an existing assertion.
      \item Treat the antecedent as an hypotheses and try to support it by backward chaining through other rules using the existing binding set.
    \end{itemize}
  \item Repeat the previous step for each antecedent, accumulating variable bindings, until,
    \begin{itemize}
      \item There is no match with any existing assertion or rule consequent using the binding set established so far. In this case, back up to the most recent match with unexplored bindings, looking for an alternative match that produces a workable binding set.
      \item There are no more antecedents to be matched. In this case, the binding set in hand supports the original hypothesis.
    \end{itemize}
  \item If all possible binding sets are desired, report the current binding set, and back up, as if there were no match.
  \item If only one possible binding set is desired, report the current binding set and quit.
  \item There are no more alternative matches to be explored at any level.
\end{itemize}
Relational Operations Support Forward Chaining

Now it is time to look at another approach to forward chaining. First, you learn how relational database operations can handle the bookkeeping required for forward chaining. Then, you learn how the relational database operations can be arranged to produce high-speed operation.

All that you need to know about relational databases in this section is introduced as you need it. If you have not studied relational databases elsewhere, and find the introduction in this section to be too brief, read the appendix, which describes relational databases in more detail.

Now consider the Parent Rule and assertions previously used to demonstrate the search-oriented approach. Here, again, is the Parent Rule.

Parent Rule

If

\[ ?x \text{ is-a horse} \]
\[ ?z \text{ is-a-parent-of } ?y \]
\[ ?y \text{ is fast} \]

then

\[ ?x \text{ is valuable} \]

Now think of the assertions as though they were part of a table. In the language of relations, the assertions are recorded in a relation, named Data, whose columns are labeled with field names—namely First, Second, and Third:

<table>
<thead>
<tr>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>is-a</td>
<td>horse</td>
</tr>
<tr>
<td>Prancer</td>
<td>is-a</td>
<td>horse</td>
</tr>
<tr>
<td>Comet</td>
<td>is-a-parent-of</td>
<td>Dasher</td>
</tr>
<tr>
<td>Comet</td>
<td>is-a-parent-of</td>
<td>Prancer</td>
</tr>
<tr>
<td>Prancer</td>
<td>is</td>
<td>fast</td>
</tr>
<tr>
<td>Dasher</td>
<td>is-a-parent-of</td>
<td>Thunder</td>
</tr>
<tr>
<td>Thunder</td>
<td>is</td>
<td>fast</td>
</tr>
<tr>
<td>Thunder</td>
<td>is-a</td>
<td>horse</td>
</tr>
<tr>
<td>Dasher</td>
<td>is-a</td>
<td>horse</td>
</tr>
</tbody>
</table>

To determine what values of \( x \) and \( y \) trigger the rule, you first determine which of the relation’s records match the first antecedent in the rule, \( ?x \text{ is-a horse} \). In the language of relations, you need to find those records whose Second field value is \( \text{is-a} \) and whose Third field value is \( \text{horse} \). Conveniently, relational database systems include an access procedure, \texttt{SELECT}, that extracts records with specified field values from one relation to produce a new relation with fewer records. You can ask \texttt{SELECT} to pick the horses out of the Data relation, for example:
SELECT Data with Second = is-a and Third = horse

The result is the new relation:

<table>
<thead>
<tr>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>is-a</td>
<td>horse</td>
</tr>
<tr>
<td>Prancer</td>
<td>is-a</td>
<td>horse</td>
</tr>
<tr>
<td>Thunder</td>
<td>is-a</td>
<td>horse</td>
</tr>
<tr>
<td>Dasher</td>
<td>is-a</td>
<td>horse</td>
</tr>
</tbody>
</table>

All you really want to know, however, is which bindings of $x$ produce matches. Accordingly, you use another relational database access procedure, PROJECT, to isolate the appropriate field:

**PROJECT Result over First**

At this point, the field named First is renamed $X$ to remind you that it consists of bindings for the $x$ variable. The result, a single-field relation, is as follows:

A1

<table>
<thead>
<tr>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
</tr>
<tr>
<td>Prancer</td>
</tr>
<tr>
<td>Thunder</td>
</tr>
<tr>
<td>Dasher</td>
</tr>
</tbody>
</table>

Next, you determine which of the records in the data relation match the second antecedent in the rule, $\exists x \text{ is-a-parent-of } y$. You need to select those records whose Second field value is is-a-parent-of. Then you project the results over the First and Third fields:

**PROJECT**

$[\text{SELECT Data with Second = is-a-parent-of}]$

over First and Third

After renaming the field named First to $X$ and the field named Third to $Y$, you have the following table:

A2

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Dasher</td>
</tr>
<tr>
<td>Comet</td>
<td>Prancer</td>
</tr>
<tr>
<td>Dasher</td>
<td>Thunder</td>
</tr>
</tbody>
</table>
Finally, you need to determine which of the records in the data relation match the third antecedent in the rule, \( \mathit{?y \text{ is fast}} \). Accordingly, you select those records whose Second field value is \( \mathit{is} \) and whose Third field value is \( \mathit{fast} \), and you project the result over the First field:

\[
\text{PROJECT} \quad \text{[SELECT Data with Second $=$ is and Third $=$ fast]} \\
\quad \text{over First}
\]

After renaming the field named First to \( Y \), reflecting the fact that the field values are possible bindings for \( y \), you have the following table:

\[
A3 \\
Y \\
Prancer \\
Thunder
\]

You now have three new relations—\( A1, A2, \) and \( A3 \)—corresponding to the three antecedents in the rule. The next question is, What bindings of \( x \) satisfy both the first and second antecedents? Or, What field values are found both in \( A1 \)'s \( X \) field and in \( A2 \)'s \( X \) field?

The \textsc{JOIN} operation builds a relation with records constructed by concatenating records, one from each of two source tables, such that the records match in prescribed fields. Thus, you can join \( A1 \) and \( A2 \) over their \( X \) fields to determine which values of \( x \) are shared. Here is the required \textsc{JOIN} operation:

\[
\text{JOIN} \ A1 \text{ and } A2 \text{ with } X = X
\]

The result is a relation in which field-name ambiguities are eliminated by concatenation of ambiguous field names with the names of the relations that contribute them:

\[
B1 \text{ (preliminary)} \\
\begin{array}{ccc}
\text{X.A1} & \text{X.A2} & Y \\
\text{Comet} & \text{Comet} & \text{Dasher} \\
\text{Comet} & \text{Comet} & \text{Prancer} \\
\text{Dasher} & \text{Dasher} & \text{Thunder}
\end{array}
\]

All you really want, of course, is to find the pairs of bindings for \( x \) and \( y \) that satisfy the first two antecedents. Accordingly, you can project the preliminary \( B1 \) relation over, say, \( \text{X.A1} \) and \( Y \), with the following result, after renaming of the fields:
B1

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Dasher</td>
</tr>
<tr>
<td>Comet</td>
<td>Prancer</td>
</tr>
<tr>
<td>Dasher</td>
<td>Thunder</td>
</tr>
</tbody>
</table>

B1 now contains binding pairs that simultaneously satisfy the first two antecedents in the rule. Now you can repeat the analysis to see which of these binding pairs also satisfy the third antecedent.

To begin, you join A3 and B1 over their Y fields to determine which values of y are shared:

JOIN A3 and B1 with Y = Y

The result is as follows:

B2 (preliminary)

<table>
<thead>
<tr>
<th>Y.A3</th>
<th>X</th>
<th>Y.B1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prancer</td>
<td>Comet</td>
<td>Prancer</td>
</tr>
<tr>
<td>Thunder</td>
<td>Dasher</td>
<td>Thunder</td>
</tr>
</tbody>
</table>

Now you project to determine the pairs of bindings for x and y that satisfy not only the first two antecedents, but also the third:

B2

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Prancer</td>
</tr>
<tr>
<td>Dasher</td>
<td>Thunder</td>
</tr>
</tbody>
</table>

At this point, you know that there are two binding pairs that simultaneously satisfy all three antecedents. Inasmuch as the then part of the rule uses only the binding of x, you project B2 over the X field:

B2

<table>
<thead>
<tr>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
</tr>
<tr>
<td>Dasher</td>
</tr>
</tbody>
</table>

Thus, the parent rule is triggered in two ways: once with x bound to Comet, and once with x bound to Dasher. In a deduction system, both binding sets can be used. In a reaction system, a conflict-resolution procedure would be required to select the next action.

The only problem with the procedure that you just learned about is that it consumes a large amount of computation. If a rule has n antecedents, then it takes n SELECT and n PROJECT operations to produce
the A relations along with \( n - 1 \) JOIN and \( n - 1 \) PROJECT operations to produce the B relations. If there happen to be \( m \) rules, and if you check out each rule whenever a new assertion is added to the data relation, then you have to perform \( mn \) SELECTS, \( m(2n - 1) \) PROJECTS, and most alarmingly, \( m(n - 1) \) expensive JOINS each time a new assertion is added. Fortunately, there is another way to search for variable bindings that does not use so many operations.

**The Rete Approach Deploys Relational Operations Incrementally**

You have just learned how to use relational operations to find binding sets, but the method described is an expensive way to do forward chaining, because a great deal of work has to be done to trigger a rule. Now you are ready to learn that the relational operations can be performed incrementally, as each new assertion is made, reducing both the total amount of work and the time it takes to trigger a rule once all the triggering assertions are in place.

Ordinarily, the word rete is an obscure synonym for net, found only in large dictionaries. In the context of forward chaining, however, the word **rete procedure** names a procedure that works by moving each new assertion, viewed as a relational record, through a rete of boxes, each of which performs a relational operation on one relation or on a few, but never on all the relations representing accumulated assertions.

The arrangement of the rete for the valuable-horse example is shown in figure 7.9.

As a new assertion is made, it becomes a single-record relation. That single-record relation is then examined by a family of SELECT operations, each of which corresponds to a rule antecedent.

In the example, the first assertion is *Comet is-a horse*. Accordingly, the following single-record relation is constructed:

<table>
<thead>
<tr>
<th>New-assertion</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>is-a</td>
<td></td>
<td>horse</td>
</tr>
</tbody>
</table>

This relation is examined by three SELECT operations:

```
SELECT new-assertion with Second = is-a and Third = horse
SELECT new-assertion with Second = is-a-parent-of
SELECT new-assertion with Second = is and Third = fast
```

Next, whenever the record in the single-record relation makes it past a SELECT operation, the single-record relation endures a PROJECT operation that picks off the field or fields that contain bindings.
Figure 7.9 The rete for a simple rule about horses with fast offspring. Here the state of the rete is captured just following the addition made in response to the first assertion. In this and other figures, the most recent changes are shown shaded.
In the example, the record makes it past the first of the three SELECT operations, whereupon the PROJECT and renaming operations produce the following:

New-assertion

<table>
<thead>
<tr>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
</tr>
</tbody>
</table>

Once a record has gone past the SELECT, PROJECT, and renaming operations, it is added to a relation associated with a rule antecedent. Each antecedent-specific relation is located at an **alpha node** created specifically for the antecedent. Each alpha-node relation accumulates all the assertions that make it through the corresponding, filterlike SELECT operation. In the example, the selected, projected, and renamed record is added to the A1 node—the one attached to the first antecedent.

The second assertion, *Prancer is a horse*, follows the first through the rete, and also ends up as a record in the relation attached to the A1 node. Then, the third assertion, Comet is a parent of Dasher, following a different route, ends up as a record in the relation attached to the A2 node.

Each addition to an alpha node’s relation inspires an attempt to join the added record, viewed as a single-record relation, with another relation. In particular, an addition to either A1’s relation or A2’s relation leads to joining of the added record, viewed as a single-record relation, with the relation attached to the other alpha node. Importantly, the JOIN operation is done with a view toward determining whether the variable binding expressed in the added record corresponds to a variable binding already established in the other relation.

In the example, the added record—the one added to A2’s relation—produces the following single-record relation:

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Dasher</td>
</tr>
</tbody>
</table>

Meanwhile, A1’s relation has accumulated two records:

A1

<table>
<thead>
<tr>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
</tr>
<tr>
<td>Prancer</td>
</tr>
</tbody>
</table>

Joining the two relations over the X field and projecting to eliminate one of the redundant X fields yields a one-record, two-field relation:
This new relation is then added to a relation attached to a **beta node**—the B12 node—so named because it is the **JOIN** of the A1 and A2 relations. B12’s relation contains a single record that records a pair of bindings for \( x \) and \( y \) that satisfies the first and second antecedents.

Thus, an addition to either A1’s relation or A2’s relation leads to a **JOIN** operation that may add one or more records to B12’s relation reflecting variable bindings that satisfy the first two rule antecedents simultaneously.

The next assertion—the fourth—**Comet is-a-parent-of Prancer**, produces the wave of activity in the rete shown by the shading in figure 7.10.

The wave starts with the addition of a second record to A2’s relation. This new record, viewed as a single-record relation, is joined to A1’s relation, producing a second record for B12’s relation.

Because it is tiresome to append the phrase, **viewed as a relation**, each time a record, viewed as a relation, is joined with another relation, let us agree to speak of joining records with a relation, even though, strictly speaking, only relations are joined.

Next, the fifth assertion, **Prancer is fast**, initiates a wave of additions to the records in the rete and leads to a record in A3’s relation. In general, an addition to A3’s relation leads to joining the added record with B12’s relation. This **JOIN** operation is done with a view toward determining whether the variable binding expressed in the added record corresponds to a variable binding already established in B12’s relation.

The result is added to B23’s relation. In this example, the **JOIN** operation is over the Y fields, and the **JOIN** operation produces—after doing a **PROJECT** to eliminate one of the redundant Y fields—an initial record for B23’s relation:

<table>
<thead>
<tr>
<th>B23</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Prancer</td>
<td></td>
</tr>
</tbody>
</table>

Projecting this new record over the X field yields a possible binding for \( x \) in the rule’s *then* part:

```
Parent-Rule Bindings B23
<table>
<thead>
<tr>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
</tr>
</tbody>
</table>
```

Thus, an addition to A3’s relation has led to joining the added record with B12’s relation. Symmetrically, of course, any new records added to B12’s
Figure 7.10 Here, the state of the rete is captured just following the additions made in response to the fourth assertion, Comet is-a-parent-of Prancer. The additions are shown shaded.
relation are joined to A3's relation. As before, the JOIN operation is done to determine whether the variable bindings expressed in the added records correspond to variable bindings already expressed in the other relation involved in the JOIN operation.

Now consider the state of the rete after you add three more assertions—*Dasher is-a-parent-of Thunder, Thunder is fast,* and *Thunder is-a horse.* A1's relation indicates that there are three horses:

```
A1
<table>
<thead>
<tr>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
</tr>
<tr>
<td>Prancer</td>
</tr>
<tr>
<td>Thunder</td>
</tr>
</tbody>
</table>
```

A2's relation indicates that Comet is a parent of two children, and Dasher is a parent of one child:

```
A2
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Dasher</td>
</tr>
<tr>
<td>Comet</td>
<td>Prancer</td>
</tr>
<tr>
<td>Dasher</td>
<td>Thunder</td>
</tr>
</tbody>
</table>
```

A3's relation indicates that Prancer and Thunder are fast:

```
A3
<table>
<thead>
<tr>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prancer</td>
</tr>
<tr>
<td>Thunder</td>
</tr>
</tbody>
</table>
```

Next, the information in the alpha-node relations is joined to form the beta-node relations:

```
B12
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Dasher</td>
</tr>
<tr>
<td>Comet</td>
<td>Prancer</td>
</tr>
</tbody>
</table>
```

```
B23
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>Prancer</td>
</tr>
</tbody>
</table>
```
Next, the ninth assertion, *Dasher is-a horse*, initiates another wave of additions to the records in the rete—the additions indicated by the shading in figure 7.11.

The first of these additions is the new record in A1’s relation. This new record is joined to A2’s relation, producing a new record in B12’s relation:

\[
\begin{array}{|c|c|}
\hline
X & Y  \\
\hline
\text{Dasher} & \text{Thunder}  \\
\hline
\end{array}
\]

But now this new B12 record is joined to A3’s relation producing a new record for B23’s relation:

\[
\begin{array}{|c|c|}
\hline
X & Y  \\
\hline
\text{Dasher} & \text{Thunder}  \\
\hline
\end{array}
\]

Projection of this new record over the X field yields another possible binding for \( x \) in the rule’s consequent:

\[
\begin{array}{|c|}
\hline
\text{Parent-Rule Bindings (increment) B23} & X  \\
\hline
\text{Dasher}  \\
\hline
\end{array}
\]

Thus, after all nine assertions are processed, the possible bindings for \( x \) in the rule’s consequent are given by the following relation:

\[
\begin{array}{|c|}
\hline
\text{Parent-Rule Bindings B23} & X  \\
\hline
\text{Comet}  \\
\text{Dasher}  \\
\hline
\end{array}
\]

As you have seen, adding a new relation produces a wavelike phenomenon that continues through the rete as long as JOIN operations produce new records. Note that all the relational operations involve only small relations containing a few assertions; they never involve the entire accumulated database of assertions.

Although the example may seem complicated, the procedures for building and using a rete are straightforward:
Figure 7.11 Here, the state of the rete is captured just following the additions made in response to the ninth assertion, *Dasher is a horse*. The most recent changes are shown shaded.
To construct a rete,

- For each antecedent pattern that appears in the rule set, create a SELECT operation that examines new assertions.
- For each rule,
  - For each antecedent,
    - Create an alpha node and attach it to the corresponding SELECT operation, already created.
- For each alpha node, except the first,
  - Create a beta node.
  - If the beta node is the first beta node, attach it to the first and second alpha nodes.
  - Otherwise, attach the beta node to the corresponding alpha node and to the previous beta node.
- Attach a PROJECT operation to the final beta node.

To use a rete,

- For each assertion, filter the assertion through the SELECT operations, passing the assertion along the rete to the appropriate alpha nodes.
- For each alpha node receiving an assertion, use the PROJECT operation to isolate the appropriate variable bindings. Pass these new bindings, if any, along the rete to the appropriate beta nodes.
- For each beta node receiving new variable bindings on one of its inputs, use the JOIN operation to create new variable binding sets. Pass these new variable binding sets, if any, along the rete to the next beta node or to the final PROJECT operation.
- For each rule, use the PROJECT operation to isolate the variable bindings needed to instantiate the consequent.

**SUMMARY**

- Rule-based systems were developed to take advantage of the fact that a great deal of useful knowledge can be expressed in simple if-then rules.
- Many rule-based systems are deduction systems. In these systems, rules consist of antecedents and consequents. In one example, a toy deduction system identifies animals.
- Deduction systems may chain together rules in a forward direction, from assertions to conclusions, or backward, from hypotheses to questions. Whether chaining should be forward or backward depends on the problem.
- Many rule-based systems are reaction systems. In these systems, rules consist of conditions and actions. A toy reaction system bags groceries.
- Reaction systems require conflict-resolution strategies to determine which of many triggered rules should be allowed to fire.
- Depth-first search can supply compatible bindings for both forward chaining and backward chaining.
- Relational operations support breadth-first search for compatible bindings during forward chaining. The rete procedure performs relational operations incrementally as new assertions flow through a rule-defined rete.

**BACKGROUND**

The rete procedure was developed by C. L. Forgy [1982].

**MYCIN** was developed by Edward Shortliffe and colleagues at Stanford University [1976].

**XCON** was developed to configure the Digital Equipment Corporation's VAX computers by John McDermott and other researchers working at Carnegie Mellon University, and by Arnold Kraft, Dennis O'Connor, and other developers at the Digital Equipment Corporation [McDermott 1982].