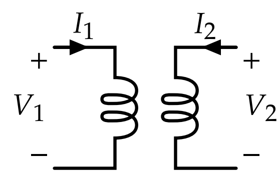
MASSACHVSETTS INSTITVTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science
6.01—Introduction to EECS I
Spring Semester, 2008

Exploration 8, Issued: Tuesday, April 3

Exploration: Modeling Motors and Generators with Transformers

A transformer is a two port element pictured below.



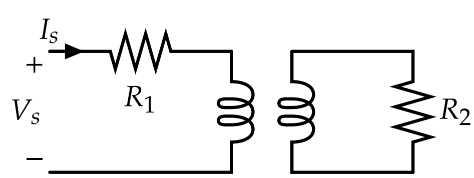
An ideal transformer can be modelled with the following equations:

$$V_2 = NV_1$$

$$I_2 = -\frac{I_1}{N}$$

Notice that these equations imply that the total power into an ideal transformer is zero. That's how a transformer gets its name: it transforms currents and voltages at side 1 to currents and voltages at side 2 without consuming any power itself.

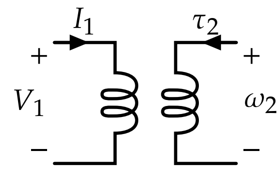
Consider a transformer with one side connected to a voltage source (V_s) through an input resistor R_1 and the other side connected to a second resistor R_2 as shown below.



Question 1: Determine an expression for $\frac{V_s}{I_s}$. Use this expression to develop an equivalent circuit that does not have a transformer. Explain how part of this equivalent circuit can be thought of as a "Thevinin Equivalent" representation for the transformer and its load resistor R_2.

In design lab 7, we investigated a simple model for a motor (Question 16). The model was a resistor in series with a voltage source that represented the effect of rotating the motor shaft. The model had some limitations. Most importantly, it only worked for one input voltage.

We can develop a better motor model using a transformer. In the transformer model of a motor, one of the transformer ports (sides) represents electrical variables and the other represents mechanical variables. Thus V_1 and I_1 represent voltage and current, respectively, as before. However, on side two, V_2 is replaced by angular velocity ω_2 and I_2 is replaced by torque τ_2. The function of the motor is to convert electrical power (left) to mechanical power (right).

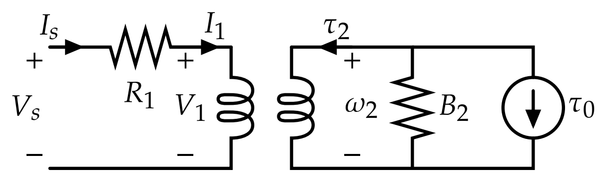


The variables for this electro-mechanical transformer are related in much the same way that they are in a purely electrical transformer, however the constant of proportionality is no longer non-dimensional.

$$\omega_2 = \alpha V_1$$

$$\tau_2 = -\frac{I_1}{\alpha}$$

To complete the motor model, we also need a resistor R_1, a mechanical resistor B_2 and a torque source τ_0 as shown below.



Question 2: Analyze the model of the motor and sketch the relation between V_s and I_s that will result. Indicate how this relation depends on the parameters R_1, B_2, and τ_0.

Question 3: Measure the relation between V_s and I_s for our Lego motor (Connect the motor to its cable and connector. The electrical pins are numbers 5 and 6 on the connector). Does this relation match the one predicted in the previous question? [Hint: the parameters of the model are different for each of three regimes, which correspond to when the motor is rotating forward, rotating backward, and not rotating.]

Question 4: Explain what the parameters R_1, B_2, and τ_0 represent physically.

Now connect the shafts of two motors so that you can use one as a motor and the other as a generator.

Question 5: Measure the relation between V_s and I_s that results when the motor is mechanically driving the generator. Is it the same as the relation when the motor was not connected to the generator? Explain.

Question 6: Drive the motor/generator with V_s and measure the voltage produced by the generator. Repeat for different values of V_s. Discuss the relation between V_s and the voltage from the generator.

Make a 60 ohm resistor by connecting four 240 ohm resistors in parallel. Place this 60 ohm resistor across the electrical output pins of the generator (pins 5 and 6).

Question 7: Measure the relation between V_s and I_s that results when the motor is mechanically driving the generator, which is driving 60 ohms. Is it the same as the relation when the generator output had no load resistor? Explain.

Question 8: Drive the motor/generator with $V_s = 10V$ and measure the voltage produced by the generator. Measure the electrical power (volts times amps) that is being supplied to the motor. Measure the electrical power that is being consumed by the 60 ohm resistor. Is power conserved? Can you account for the difference between input and output power quantitatively?