Major topics of 6.01

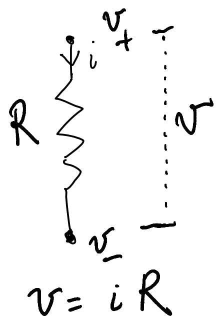
- Controlling complexity
  - abstraction and modularity
- Interacting with the real world
  - models
- Coping with error and incomplete information
  - reasoning about uncertainty

PCAP framework for Python

	Procedures	Data
Primitives	+, *, ==	numbers, strings
Combination	if, f(g(x))	lists, objects
Abstraction	def	ADTs, classes
Patterns	higher-order fns	polymorphism, inheritance

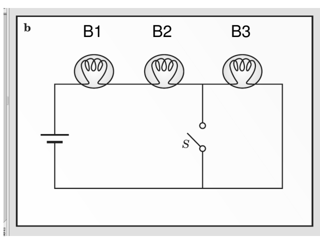
PCAP framework for signals and systems

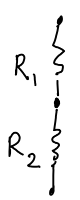
Primitives	signal
Combination	adder, gain, delay
Abstraction	system function
Patterns	feedback



Ideal linear resistor model

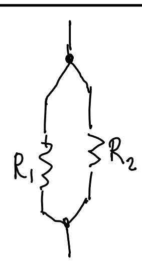
Example due to Carl Wieman





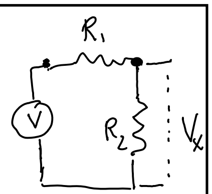
Series combination

$$R = R_1 + R_2$$



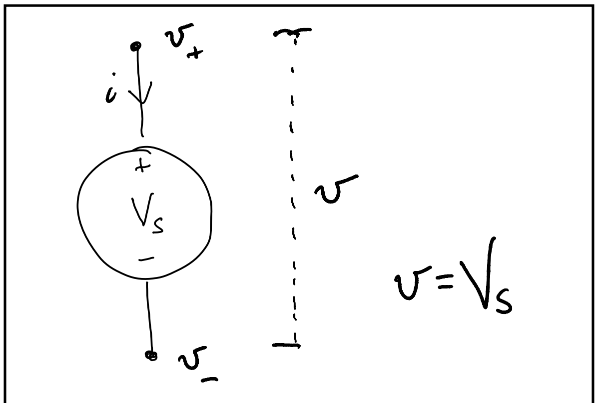
Parallel combination

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

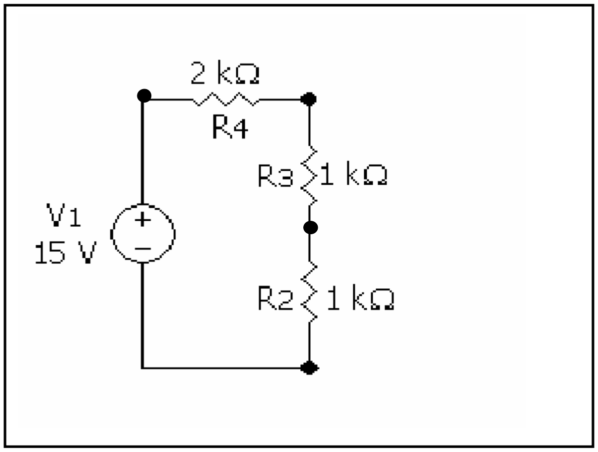


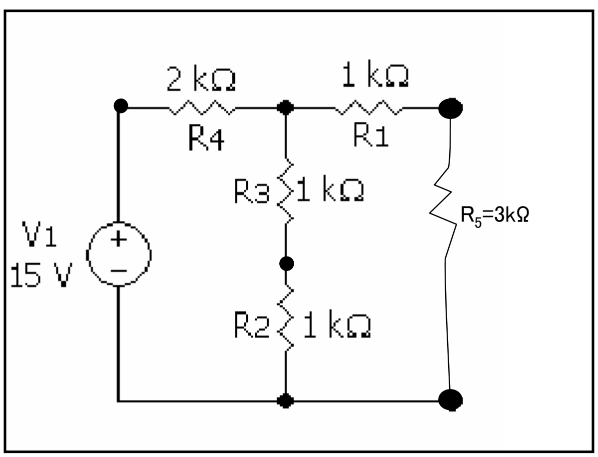
Voltage divider

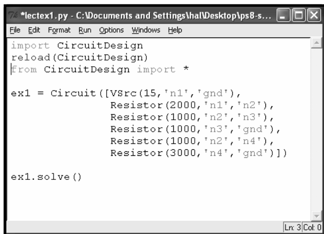
$$V_x = V \cdot \frac{R_2}{R_1 + R_2}$$



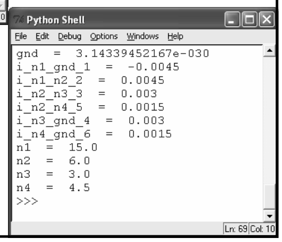
Ideal independent voltage source





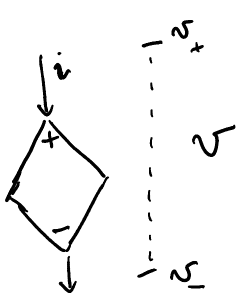


Using the circuit solver



Different styles of computational models

- Functional models
  - Sytems are described as collections of mathematical functions with inputs and outputs
- Object models
  - Things are described as collections of objects that have behavior and internal state
- Constraint models
  - Systems are described as collections of the constrains that must hold among their components. There are no “inputs” and “outputs”, just constraints that must be satisfied.



$v = f$[various circuit quantities]

* For 6.01, we’ll assume f is a linear function.

Dependent voltage source

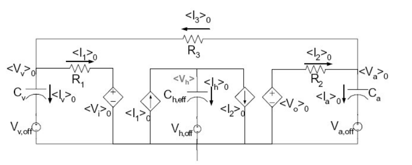
PCAP framework for circuits

Primitives	Resistors, sources, 2-terminal devices (assume all linear)
Combination	Wire parts together at the terminals
Abstraction	
Patterns	

"Modeling and Monitoring of Cardiovascular Dynamics in the Intensive Care Unit"
Tushar Parlikar, Thomas Heldt, George Verghese, 2005

- For the SPCVM, applying the cycle-averaging technique, we obtain a circuit with dependent voltage and current sources:

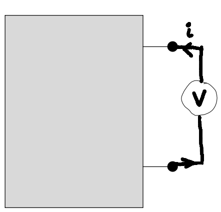
V_a ~ CVP
V_v ~ ABP
V_h ~ VP



$$\langle V_i(t)\rangle_0 = \langle q_1(t)V_h(t)\rangle_0 + \langle (1-s_1(t))V_1(t)\rangle_0 \approx \langle s_1(t)\rangle_0 \langle V_h(t)\rangle_0 + (1-\langle s_1(t)\rangle_0)\langle V_v(t)\rangle_0 + K_i$$

$$\langle V_o(t)\rangle_0 \approx \langle s_D(t)\rangle_0 \langle V_h(t)\rangle_0 + (1-\langle s_D(t)\rangle_0)\langle V_a(t)\rangle_0 + K_0$$

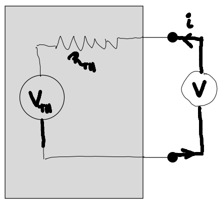
1-port



Apply a voltage and measure the current. The 1-port is completely described by the relation of the between the voltage and the current. **It doesn't matter what's in the box, so long as the relation holds.**

Analogy with software: An abstract data type is described by its operations.

Thévenin model

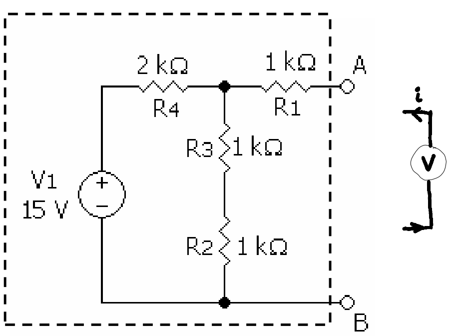


Any 1-port made up of linear resistors and sources, when viewed from the terminals, is **completely electrically equivalent** to a network composed of a single resistor and a single voltage source.

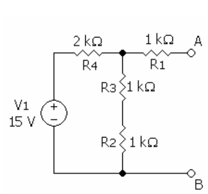
$$v = V_{TH} + iR_{TH}$$

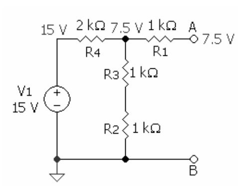
Open circuit voltage

Resistance seen from the terminals what all independent sources are suppressed

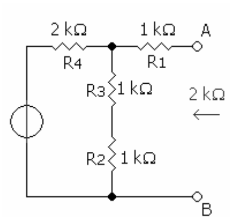


Example: From Wikipedia

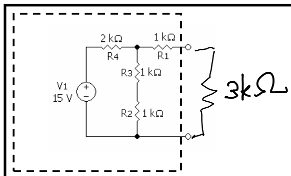


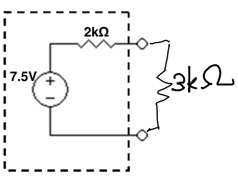


The voltage at the terminals is 7.5V, so V_{TH} is 7.5V



The resistance seen from the terminals is 2kΩ, so R_{TH} is 2kΩ

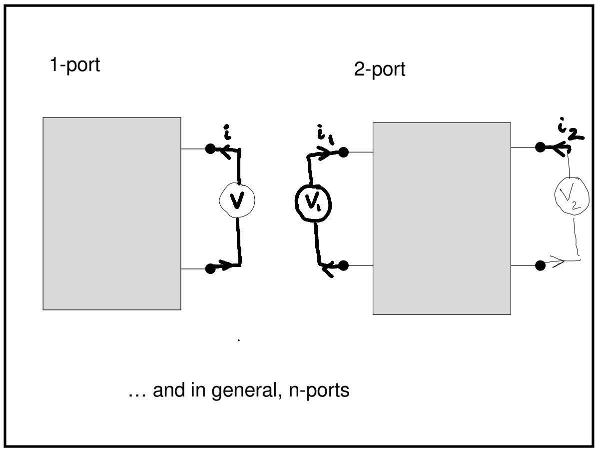




$$v = 7.5 \times \frac{3}{3+2} = 7.5 \times \frac{3}{5} = 4.5$$

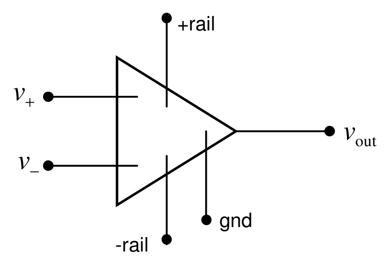
PCAP framework for circuits

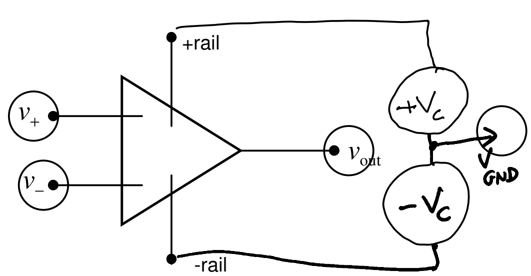
Primitives	Resistors, sources, 2-terminal devices (assume all linear)
Combination	Wire parts together at the terminals
Abstraction	1-port (Thévenin model)
Patterns	



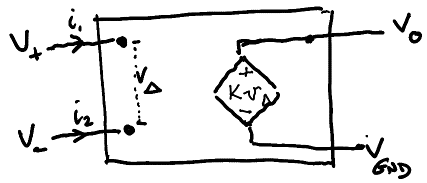
Operational amplifier (op-amp)

5-terminal device





Ideal op-amp model



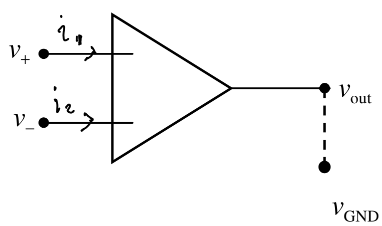
$$V_o - V_{GND} = K(V_+ - V_-)$$

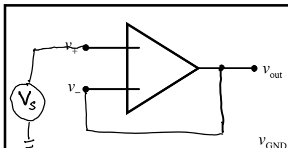
$$i_1 = i_2 = 0$$

K might be around 1,000

$$V_o - V_{GND} = K(V_+ - V_-)$$

$$i_1 = i_2 = 0$$





$$v_{\text{out}} = K(v_+ - v_-)$$

$$v_{\text{out}} = K(V_S - v_{\text{out}})$$

$$v_{\text{out}} = KV_S - Kv_{\text{out}}$$

$$v_{\text{out}} + Kv_{\text{out}} = KV_S$$

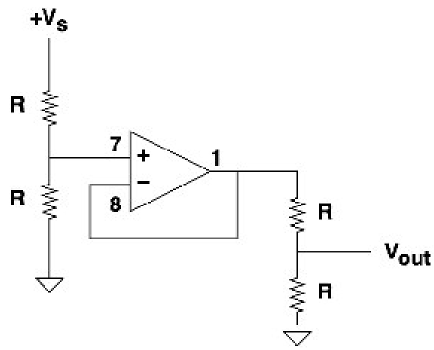
$$v_{\text{out}}(1+K) = KV_S$$

$$\frac{v_{\text{out}}}{V_S} = \frac{K}{1+K}$$

$$\frac{v_{\text{out}}}{V_S} \approx 1$$

Voltage follower (or buffer)

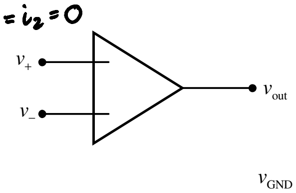
Using a buffer to cascade voltage dividers in lab last week



An even simpler op-amp model (K infinite)

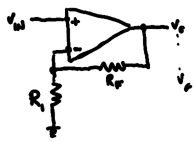
$V_0 - V_{GND} = K(v_+ - v_-)$

$i_1 = i_2 = 0$



- Draws no current, that is, i1=i2=0
- If K is very large, and v_{out} is finite, then v+=v-

Inverting amplifier

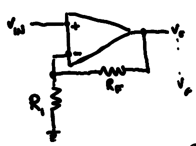


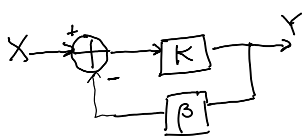
$$\frac{v_o - v_-}{R_F} = \frac{v_-}{R_1}$$

$$v_- = v_+ = v_{in}$$

- $\frac{v_o - v_{in}}{R_F} = \frac{v_{IN}}{R_1}$
- $R_1 v_o - R_1 v_{in} = R_F v_{IN}$
- $R_1 v_o = (R_1 + R_F) v_{IN}$
- $\frac{V_o}{v_{IN}} = \frac{R_F + R_1}{R_1}$

Non-inverting amplifier

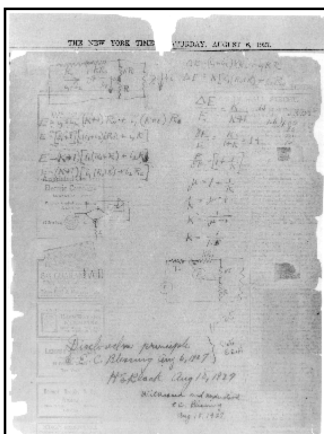




$$\frac{V_o}{v_{IN}} = \frac{R_F + R_1}{R_1}$$

$$\frac{Y}{X} = \frac{K}{1 + \beta K} \approx \frac{1}{\beta} \quad K >> 0$$

$$\beta = \frac{R_1}{R_1 + R_F}$$


THE NEW YORK TIMES ... TUESDAY, AUGUST 6, 1927.

H. S. Black Aug 1?, 1927

Harold S. Black (1898-1983)
Inventor of the negative feedback amplifier (1927)