MASSACHVSETTS INSTITVTE OF TECHNOLOGY

Department of Electrical Engineering and Compvter Science

6.01 Introdvction to EECS 1

Spring Term, 2008

Week 7 Lecture

March 18, 2008

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**Introduction to Electric Circuits**

**Primitives:**
- **Voltage**
- **Current**
- **Circuit Elements**

**Means of Combination:**
- **Circuit Constraints (KCL, KVL), describing wiring**

**Means of Abstraction:**
- **Thevenin**
- **Norton**

**Common Patterns:**
- **Series**
- **Parallel**
- **Voltage Divider**
- **Current Divider**

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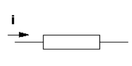
**Two primitive notions with respect to a circuit element:**




**André-Marie Ampère**

André-Marie Ampère (1775-1836)

**Alessandro Volta**

Alessandro Giuseppe Antonio Anastasio Volta (1745-1827)

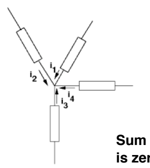
- **Current is the flow of charge (electrons) through the element. Current is measured in <u>Amperes</u>. We will usually refer to milliamperes. (mA)**

- **Voltage is the electromotive force pushing the electrons through the element. The voltage appears 'across' the element. Voltage is also called 'potential' because it represents potential energy change of the charges from one end of the element to the other. The unit of potential is the <u>Volt.</u>**

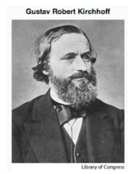
**Conservation laws govern current and voltage**

**Conservation of current: is also called KCL,**

**Or Kirchhoff's Current Law**



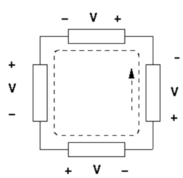

$$\sum_k i_k = 0$$




**1824-1887**

**Sum of all currents entering a node is zero, meaning whatever current enters a node must also leave the node. Current does not build up, but must go in circles.**

**KVL is Kirchhoff's Voltage Law, and it says that voltage really is a potential that is single valued. The sum of voltages around a loop must add to zero. So if you take one node as 'datum' or 'ground', every other node has a uniquely defined potential (voltage)**

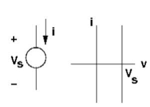



$$\sum_k v_k = 0$$

**Primitive Circuit Elements**

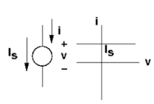
**Voltage Source: Fixes its terminal voltage, independent of current**

**(Sort of like a battery, but more about that later)**



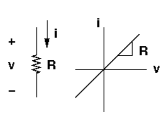

**Curent source: Fixes its terminal current, independent of voltage**

**(sort of like lightning, but you don't want to fiddle with that…)**

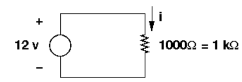



**Resistance: Has a fixed ratio of voltage to current:**

**The unit if resistance is the <u>Ohm,</u> a volt/ampere.**

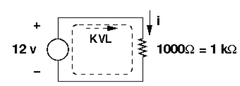



$$V = RI$$




**Here is a simple problem: what current will flow in the resistor?**

**What current will flow in the source?**




**Around the KVL loop:** $-V_s + V_r = 0$

**So current is:** $i = \frac{V_s}{R} = \frac{12v}{1k\Omega} = 12mA$

**One more thing about this:**

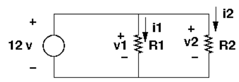
**Power is** $P = V \times I = I^2 R = \frac{V^2}{R}$

**For 12 v and 1k, that is 144 mW**

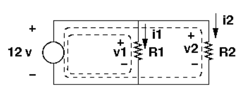
**But if R=100 Ohms, it is 1.44 W.**

**How would a 1/4 watt resistor handle this?**

Here is a second problem, with two resistors:




KVL can be used around two loops:

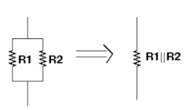



$$V_1 = V_2 = V_s = 12V$$

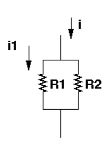
And then KCL yields, for current OUT OF the source:

$$I_s = I_1 + I_2 = V_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

This is a parallel resistor combination:



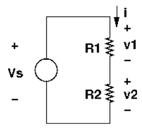

And the same combination gives us
the current divider:




$$v = iR_1 \parallel R_2 = \frac{i}{\frac{1}{R_1} + \frac{1}{R_2}} = i\frac{R_1R_2}{R_1 + R_2}$$

$$i_1 = \frac{v}{R_1} = i\frac{R_2}{R_1 + R_2}$$

**Series Combination:**



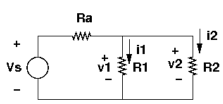

$$V_s = V_1 + V_2 = i\left(R_1 + R_2\right)$$

$$R_s = R_1 + R_2$$

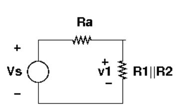
**And the associated voltage divider relationship:**

$$V_2 = R_2 i = V_s \frac{R_2}{R_1 + R_2}$$

**A simple problem: what are the currents?**




**As far as voltages are concerned, this is the same circuit**

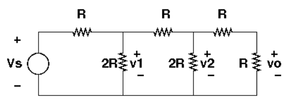



$$v_1 = V_s \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_a} = V_s \frac{R_1 R_2}{R_a R_1 + R_a R_2 + R_1 R_2}$$

$$i_1 = \frac{v_1}{R_1} = V_s \frac{R_2}{R_a R_1 + R_a R_2 + R_1 R_2}$$

$$i_2 = \frac{v_1}{R_2} = V_s \frac{R_1}{R_a R_1 + R_a R_2 + R_1 R_2}$$

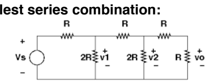
So what about a more complicated problem? What is the output voltage of this one?

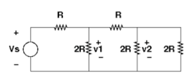


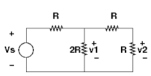

Here is the formulation of the problem in circuit constraints:

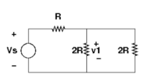
$$\frac{1}{R}(v_1 - V_s) + \frac{1}{2R}v_1 + \frac{1}{R}(v_1 - v_2) = 0$$
$$\frac{1}{R}(v_2 - v_1) + \frac{1}{2R}v_2 + \frac{1}{R}(v_2 - v_o) = 0$$
$$\frac{1}{R}(v_o - v_2) + \frac{1}{R}v_o = 0$$

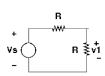
It is not that bad: look at serial and parallel combinations to get to the Simplest series combination:



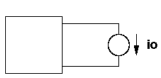



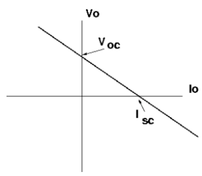




$$v_o = v_2 \frac{R}{R+R} = \frac{1}{2}v_2$$
$$v_2 = v_1 \frac{R}{R+R} = \frac{1}{2}v_1$$
$$v_1 = V_s \frac{R}{R+R} = \frac{1}{2}V_s$$
$$v_0 = \frac{1}{2}v_2 = \frac{1}{4}v_1 = \frac{1}{8}V_s$$

Now what would it look like if you were to allow it to supply some current? Consider what might happen with a current source connected to the output:



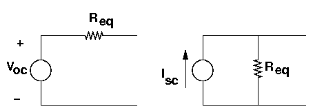
It is, after all, a linear circuit, as we recognized by Thevenin




So output voltage is a linear function of current

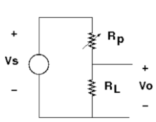

1857-1926

The behavior of any circuit of this type can be represented by either the Thevinin or Norton equivalents (which are equivalent)




It is perhaps remarkable, and can be proven, that any circuit made of sources and other linear components may be represented this way. We won't <u>prove</u> it, but we will <u>use</u> this fact.

So how do we use these things? Consider a photo detector:




In dark, $R_p = 10k\Omega \quad v_o = \frac{1}{2}V_s$

In light, $R_p = 1k\Omega \quad v_o = \frac{10}{11}V_s$

You will use a photoresistor in the lab: it may not have exactly these values

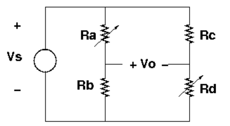
If Vs=10v,

Dark Vo = 5 v

Light Vo = 9.1 v

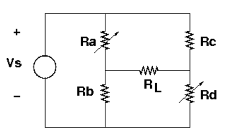
And you can detect this

**A very useful resistive circuit: Wheatstone Bridge**

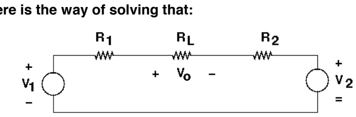



$$V_0 = V_s\left(\frac{R_b}{R_a + R_b} - \frac{R_d}{R_c + R_d}\right)$$

**How to handle this one? Voltage across $R_L$**




**Here is the way of solving that:**




$$V_1 = V_s\frac{R_b}{R_a + R_b}$$

$$V_2 = V_s\frac{R_d}{R_c + R_d}$$

$$R_1 = R_a \parallel R_b$$

$$R_2 = R_c \parallel R_c$$

$$V_o = (V_1 - V_2)\frac{R_L}{R_L + R_1 + R_2}$$