Introduction to Electric Circuits

Primitives:
- Voltage
- Current

Circuit Elements

Means of Combination:
- Circuit Constraints (KCL, KVL), describing wiring

Means of Abstraction:
- Thevenin
- Norton

Common Patterns:
- Series
- Parallel
- Voltage Divider
- Current Divider

Two primitive notions with respect to a circuit element:

- Current is the flow of charge (electrons) through the element. Current is measured in Amperes. We will usually refer to milliamperes (mA).

- Voltage is the electromotive force pushing the electrons through the element. The voltage appears across the element. Voltage is also called ‘potential’ because it represents potential energy change of the charges from one end of the element to the other. The unit of potential is the Volt.
Conservation laws govern current and voltage

Conservation of current: is also called KCL, or Kirchhoff’s Current Law

\[ \sum i_k = 0 \]

Sum of all currents entering a node is zero, meaning whatever current enters a node must also leave the node. Current does not build up, but must go in circles.

1824-1887

KVL is Kirchhoff’s Voltage Law, and it says that voltage really is a potential that is single valued. The sum of voltages around a loop must add to zero. So if you take one node as ‘datum’ or ‘ground’, every other node has a uniquely defined potential (voltage)

\[ \sum v_k = 0 \]

Primitive Circuit Elements

Voltage Source: Fixes its terminal voltage, independent of current
(Sort of like a battery, but more about that later)

Current source: Fixes its terminal current, independent of voltage
(sort of like lightning, but you don’t want to fiddle with that…)

Resistance: Has a fixed ratio of voltage to current:
The unit if resistance is the Ohm, a volt/ampere.

\[ V = RI \]
Here is a simple problem: what current will flow in the resistor? What current will flow in the source?

Around the KVL loop:

\[ V_s + V_r = 0 \]

So current is:

\[ i = \frac{V_s}{R} = \frac{12\text{v}}{1\text{k} \Omega} = 12\text{mA} \]

One more thing about this:

Power is

\[ P = V \times I = I^2 R = \frac{V^2}{R} \]

For 12 v and 1k, that is 144 mW.
But if R=100 Ohms, it is 1.44 W.
How would a 1/4 watt resistor handle this?
Here is a second problem, with two resistors:

\[ V_1 = V_2 = V_s = 12 \text{V} \]

And then KCL yields, for current OUT OF the source:

\[ I_s = I_1 + I_2 = V_s \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

This is a parallel resistor combination:

\[ R_{1||R_2} = \frac{R_1 R_2}{R_1 + R_2} \]

And the same combination gives us the current divider:

\[ v = iR \parallel R_l = \frac{i}{R_l} + \frac{1}{R_1} = \frac{R R_l}{R_1 + R_l} \]

\[ i = \frac{V}{R_l} - \frac{R_1}{R_1 + R_2} \]
Series Combination:

\[ V_s = V_1 + V_2 = i(R_1 + R_2) \]

And the associated voltage divider relationship:

\[ V_s = R_2i = \frac{V_s}{R_1 + R_2} \]

A simple problem: what are the currents?

As far as voltages are concerned, this is the same circuit:

\[ v_1 = \frac{V_s}{R_1 + R_2} = i \left( \frac{R_1}{R_1 + R_2} \right) \]

\[ i_1 = \frac{V_s}{R_1 + R_2} = i \left( \frac{R_1}{R_1 + R_2} \right) \]

\[ i_2 = \frac{V_s}{R_1 + R_2} = i \left( \frac{R_1}{R_1 + R_2} \right) \]
So what about a more complicated problem? What is the output voltage of this one?

Here is the formulation of the problem in circuit constraints:

\[
\frac{1}{R}(v_1 - v_s) + \frac{1}{2R}v_1 + \frac{1}{R}(v_2 - v_1) = 0
\]

\[
\frac{1}{R}(v_1 - v_s) + \frac{1}{2R}v_1 + \frac{1}{R}(v_2 - v_1) = 0
\]

\[
\frac{1}{R}(v_1 - v_2) + \frac{1}{R}v_1 = 0
\]

It is not that bad: look at serial and parallel combinations to get to the Simplest series combination:

\[
v_0 = v_2
\]

\[
v_2 = v_1
\]

\[
v_0 = v_2
\]

Now what would it look like if you were to allow it to supply some current? Consider what might happen with a current source connected to the output:
It is, after all, a linear circuit, as we recognized by Thevenin. So output voltage is a linear function of current.

The behavior of any circuit of this type can be represented by either the Thevenin or Norton equivalents (which are equivalent).

It is perhaps remarkable, and can be proven, that any circuit made of sources and other linear components may be represented this way. We won't prove it, but we will use this fact.

So how do we use these things? Consider a photo detector:

You will use a photoresistor in the lab; it may not have exactly these values.

If \( V_s = 10 \text{v} \),

Dark \( V_o = 5 \text{ v} \)

Light \( V_o = 9.1 \text{ v} \)

And you can detect this.
A very useful resistive circuit: Wheatstone Bridge

\[ V_o = V_s \left( \frac{R_c}{R_c + R_d} - \frac{R_d}{R_d + R_c} \right) \]

How to handle this one? Voltage across \( R_L \)

\[ V_1 = V_s \left( \frac{R_a}{R_a + R_b} \right) \]
\[ V_2 = V_s \left( \frac{R_d}{R_d + R_c} + R_a \right) \]
\[ R_1 = R_a \parallel R_b \]
\[ R_2 = R_c \parallel R_c \]
\[ V_o = V_1 - V_2 \left( \frac{R_c}{R_c + R_c + R_c} \right) \]

Here is the way of solving that: