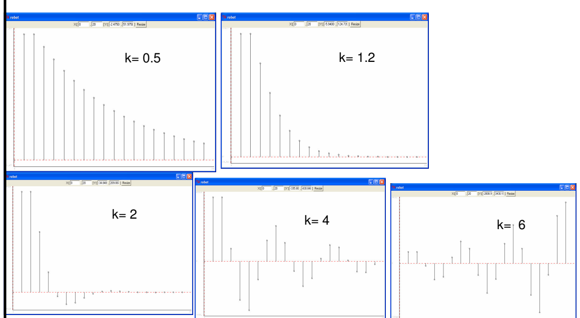
6.01 lecture notes
March 4

Modeling and abstraction with signals and linear systems

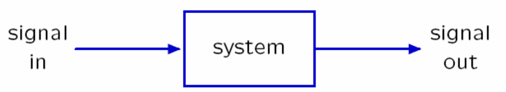
Robot meets wall



Big themes of 6.01

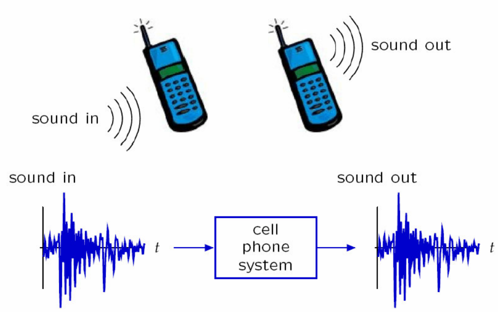
1. Controlling complexity – abstraction and modularity
2. Interacting with physical systems – models
3. Coping with error and incomplete information

Signal and system abstraction



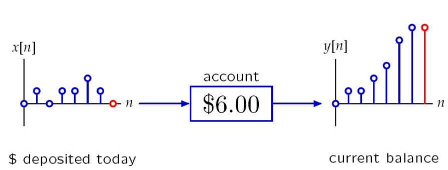
Describe a system by the way it transforms sequences of inputs in to sequences of outputs

Example: Cell Phone System



Example: Bank Account

Transactions (deposits/withdrawals) recorded daily (DT)



Example: Bank Account

Withdrawals are negative deposits.

$ deposited today

current balance

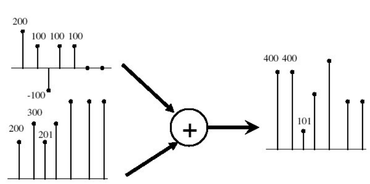
PCAP framework for Python

	Procedures	Data
Primitives	+, *, ==	numbers, strings
Combinati on	if, f(g(x))	lists, objects
Abstraction	def	ADTs, classes
Patterns	higher-order fns	polymorphism, inheritance

Adder

$y[n] = x_1[n] + x_2[n]$

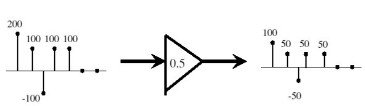
$y = x_1 + x_2$

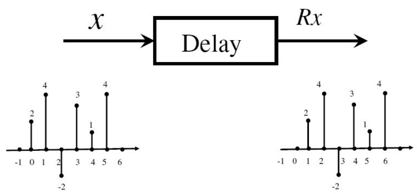


Gain

$y[n] = k \cdot x[n]$

$y = k \cdot x$

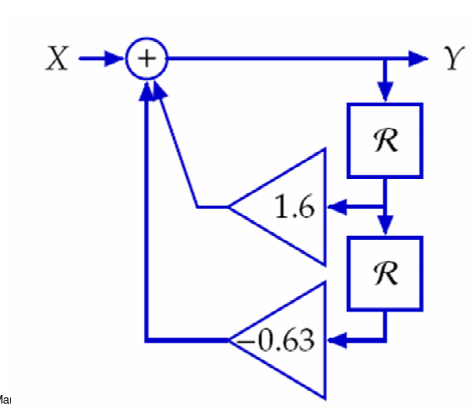




$$y[n] = x[n-1]$$

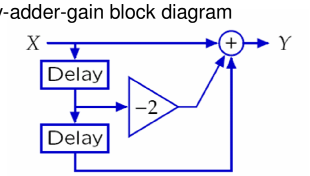
$$y = Rx$$

R is called the delay operator



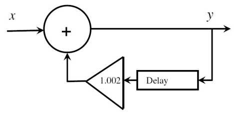
Quiz

Find the operator equation and the difference equation corresponding to this delay-adder-gain block diagram



Three representations

Block diagram: good for understanding signal flow



Difference equation: good for computing numerical outputs to given numerical inputs

$$y[n] = 1.002\,y[n-1] + x[n]$$

Operator equation: good for doing algebraic manipulation and analysis

$$y = 1.002Ry + x$$

Linear time-invariant systems (LTI)

- As block diagrams: Build from adders, (constant) gains, and delays
- As difference equations: Linear with constant coefficients

$$a_0 y[n] + a_1 y[n-1] + \cdots a_k y[n-k]$$
$$= \quad b_0 x[n] + b_1 x[n-1] + \cdots b_j x[n-j]$$

- As operator equations: Built from addition, multiplication by constants, and “multiplication” by R

$$a_0 y + a_1 Ry + \cdots a_k R^k y$$
$$= \quad b_0 x + b_1 Rx + \cdots b_j R^j x$$

Quiz

- Write the operator equation corresponding to the following difference equation

$$3y[n] + 4y[n-1] - 7y[n-2] = 10x[n] + 6x[n-2] + 8x[n-3]$$

Quiz

- Write the operator equation corresponding to the Fibonacci equation

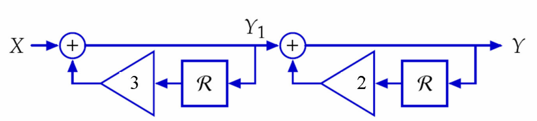
$$y[n] = y[n-1] + y[n-2] = x[n]$$

The big fact

- The operator algebra mirrors the behavior of the system, so we can reason about combining systems by doing algebra.
- This is captured by the idea of a **system function**

$$H = \frac{y}{x}$$

- This is, the system can be represented by the ratio of output y to input x as expressed by the operator equation.



$$H = \frac{y}{x} = \frac{y}{w} \cdot \frac{w}{x} = \frac{1}{1-2R} \cdot \frac{1}{1-3R} = \frac{1}{1-5R+6R^2}$$

$$y - 5Ry + 6R^2 y = x$$

$$y[n] - 5y[n-1] + 6y[n-2] = x$$

PCAP framework for signals and systems

Primitives	signal
Combination	adder, gain, delay
Abstraction	system function
Patterns	

s

Robot motion: $d[n] = d[n-1] - \delta v[n-1]$

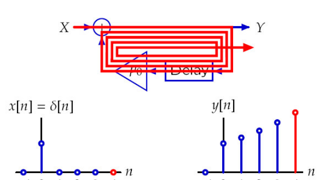
controller: $v[n] = k\left(d[n-1] - s\right)$

Put controller eqn in the form of a DE $v[n] = k\left(d[n-1] - sx[n]\right)$

where x[n]=1 for all n

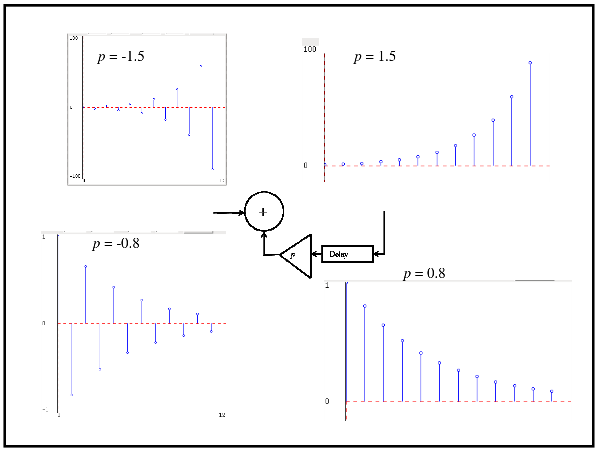
$$H = \frac{d}{x} = \frac{d}{v} \cdot \frac{v}{x}$$

$$H = \frac{y}{x} = \frac{1}{1 - p_0 R}$$



y gets multiplied by p_0 each time around the cycle. So the result is an exponential: $y[n]=p^n$

p_0 is called the **pole** of the system



The general picture: explanation next week

- The system function can be written in the form

$$\frac{\text{polynomial in } R}{(1-p_1R)(1-p_2R)\cdots(1-p_kR)}$$

- The p's are the poles
- The poles are in general *complex numbers*
- The positions of the poles in the complex plane determine the stability and oscillation of the system's response