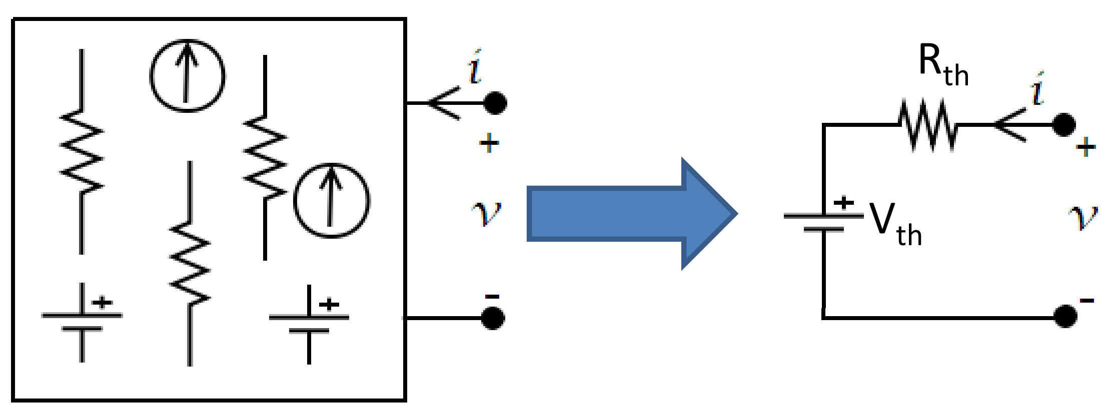
6.01 Review Session

Thevenin Equivalent Circuits

What is Thevenin's Theorem?

- Any electrical network comprised of a combination of **linear** circuit elements (i.e. **voltage sources, current sources, resistors**) can be replaced by a single voltage source, $\mathbf{V_{th}}$, and a single resistor, $\mathbf{R_{th}}$.



How can we get away with this?

- Linearity

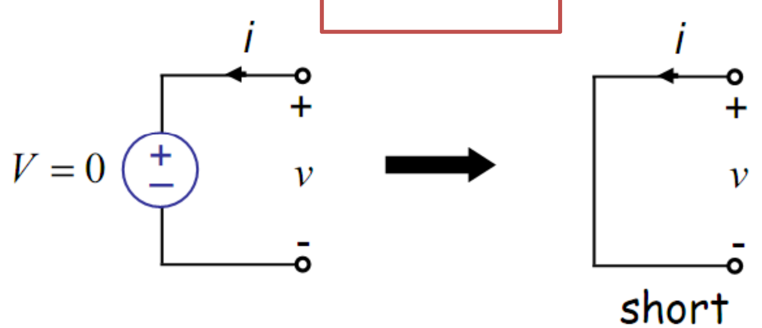
$$x_1 \rightarrow \boxed{F(x_1)} \rightarrow y_1$$
$$x_2 \rightarrow \boxed{G(x_2)} \rightarrow y_2$$

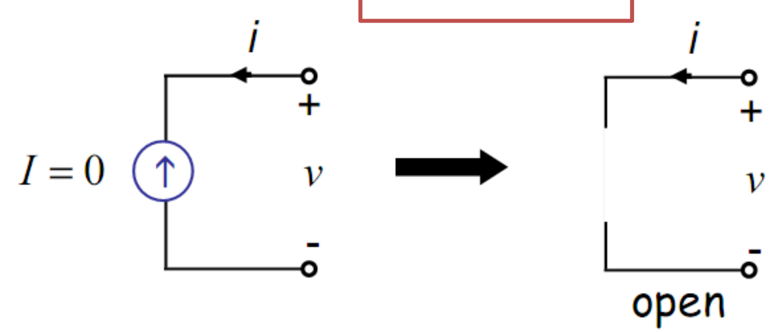
$$\Rightarrow$$

$$\alpha x_1 + \beta x_2 \rightarrow \boxed{H(x_1, x_2)} \rightarrow \alpha y_1 + \beta y_2$$

- Superposition
  - The output of a circuit comprised of **independent** sources is determined by summing the responses to each source alone.

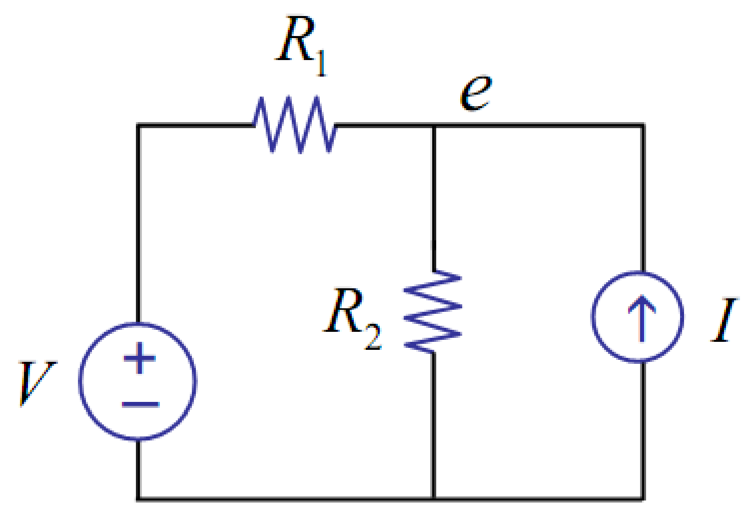
$\boxed{V = IR}$ Currents/Voltages must be finite. $\boxed{I = V/R}$





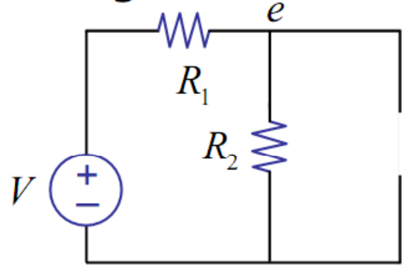
Example #1

Solve for voltage at e.



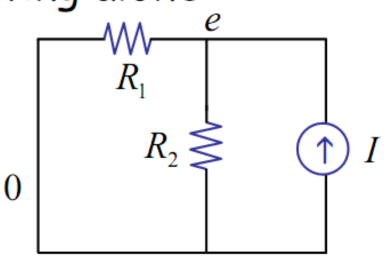
Solution

V acting alone



$I = 0$ $\quad e_V = \frac{R_2}{R_1 + R_2} V$

I acting alone



$V = 0$ $\quad e_I = \frac{R_1 R_2}{R_1 + R_2} I$

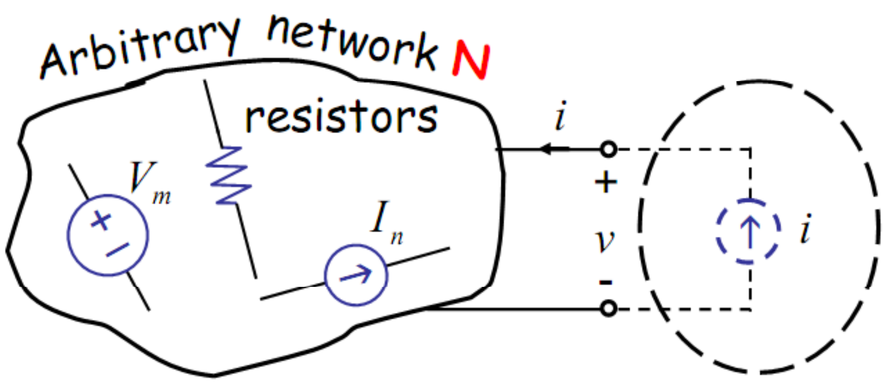
sum ⟶ superposition

$$e = e_V + e_I = \frac{R_2}{R_1 + R_2} V + \frac{R_1 R_2}{R_1 + R_2} I$$

$$\boxed{e = a_1 V_1 + a_2 V_2 + \ldots + b_1 I_1 + b_2 I_2 + \ldots}$$

Linear!

Consider



By superposition

$$v = \sum_m \alpha_m V_m + \sum_n \beta_n I_n + Ri$$

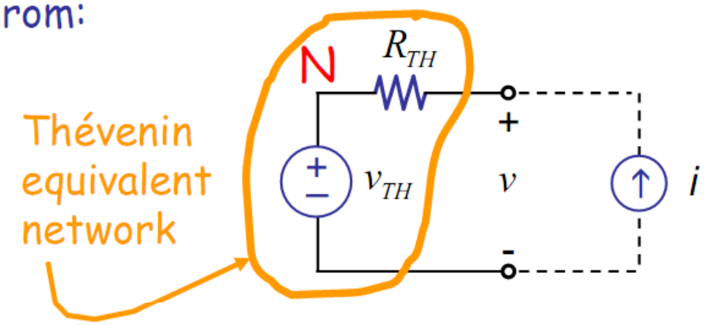
no units | resistance units | also independent of external excitement & behaves like a resistor

By setting

$\forall_n I_n = 0,\ i = 0$	$\forall_m V_m = 0,\ i = 0$	All: $\forall_n I_n = 0,\ \forall_m V_m = 0$

independent of external excitation and behaves like a voltage "v_{TH}"

"Arbitrary network N" is indistinguishable from:

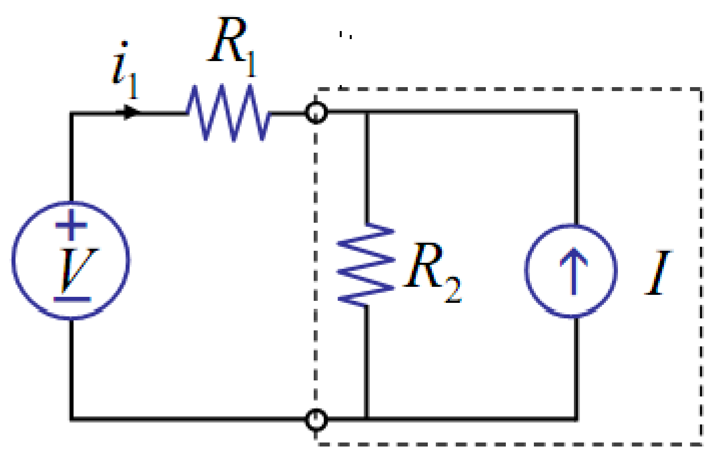


v_{TH} ⟶ open circuit voltage at terminal pair (a.k.a. port)

R_{TH} ⟶ resistance of network seen from port (V_m's, I_n's set to 0)

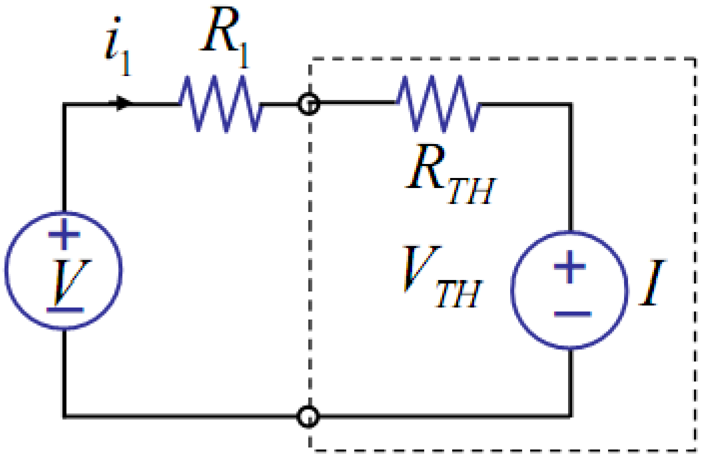
$$\boxed{v = v_{TH} + R_{TH} i}$$

Example #2



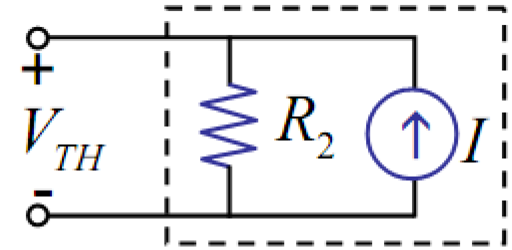
Solve for the Thevenin equivalent of the boxed area.

Solution



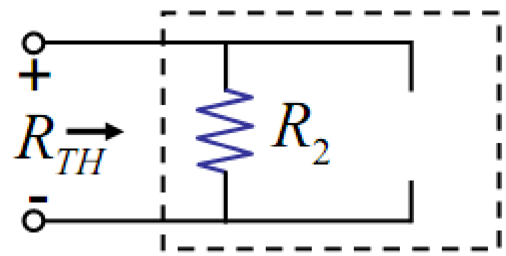
V_{TH} :

$$V_{TH} = IR_2$$



R_{TH} :

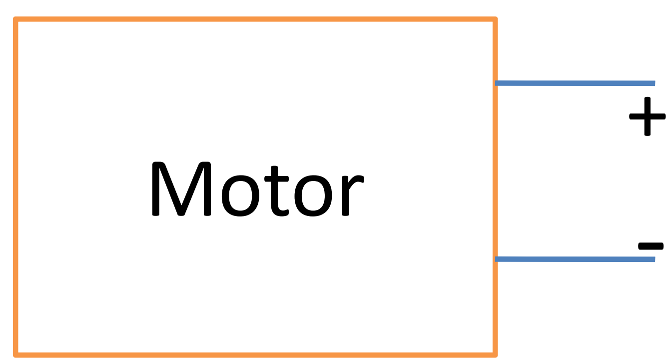
$$R_{TH} = R_2$$



$$i_1 = \frac{V - V_{TH}}{R_1 + R_{TH}}$$

Example #3

Using a multimeter, you measure the resistance of the motor (while it's off) to be 10 Ohms. Then, you measure the current through the running motor (15 V power supply) to be 100 mA. Draw the Thevenin equivalent circuit for the motor.



Solution

Because we measured the resistance at the motor terminals without any sources, the Thevenin resistance value is R_{th} = 10 Ohms. The 100 mA of current we measured implies that there is a 1 V potential drop across R_{th}. Therefore, there must be a back-EMF of 14 V to produce this potential drop. This gives us a V_{th} of 14 V for the running motor.