6.01 Review Notes
Probability and State Estimation

Probability

a. Let the sum of two dice be a random variable S. What values can S take on?

   2-12

b. Now consider the random variable E which represents whether the two dice have the same number showing. What values can E have?

   True or False

c. Let A be a random variable representing some event, and B be a random variable representing some other event. If someone tells you that for all possible values a for A and b for B,

$$P(A = a, B = b) = P(A = a)P(B = b),$$

   what does that tell you about the two events?

   The two events are independent.

d. Are S (the sum of the dice) and E (whether the dice are equal) independent events? Why or why not?

   No, they are not independent. $P(S = 5) \neq 0.0$ and $P(E = True) \neq 0.0$, but $P(S = 5, E = True) = 0.0$.

e. Consider a burglar alarm. The alarm can be set off by either a burglar trying to break into the house, or some other event, such as an earthquake, or a helium balloon bobbing around the house. What are the random variables in this scenario? What are their values?

   The random variables are whether there's a burglar, earthquake or balloon, and whether or not the alarm is going off. They are all binary.

f. Now imagine that we really only care about whether or not there is a burglar. We'll ignore all the other causes. Let A be the random variable representing whether the alarm goes off or not. Let B be the random variable for whether or not there is a burglar. Construct a plausible conditional probability table for $P(A|B)$.

   Where did all the other causes end up?

$P(A\|B)$		A	
		True	False
B	True	0.9	0.1
	False	0.05	0.95

   The non-burglar causes are all summed up in the $P(A = True|B = False)$.

g. Imagine that the alarm company has given you the conditional probability table above for $P(A|B)$ (based on experimentation), and your neighborhood watch association has kept statistics and determined that a house in your neighborhood has probability 0.1% of getting burgled over a 24 hour span. Given that the alarm is going off, how can you compute the probability that the house is being burgled?

$$P(B = True|A = True) = \frac{P(A = True|B = True)P(B = True)}{P(A = True)}$$

   We can get $P(A = True)$ by summing out all possible values of B, so:

$$P(B = True|A = True) = \frac{P(A = True|B = True)P(B = True)}{\sum_{b \in T,F} P(A = True|B = b)P(B = b)}.$$

   For this case, the probability that the house is being burgled, given that the alarm is going off, is 0.018 (compared to 0.001 with no information about the alarm). On the other hand, the probability that it is *not* being burgled, given that the alarm is going off, is 0.982.

State Estimation

State Transitions

a. Consider a robot in a toroid hallway with 5 states. The robot has two actions, **left** and **right**. This robot has two difficulties. First, it has a faulty drive system, so sometimes it doesn't actually drive when it is told to. Second, the hallway has a sinkhole at state 2, which tends to draw the robot towards it.

If the robot takes a **left** or **right** action, it moves appropriately with probability 0.6. With probability 0.1, it stays in the current state. With probability 0.2, it doesn't actually drive, but just slides one state toward the sinkhole, and with probability 0.1, it slides two states toward the sinkhole. It will never slide past or out of the sinkhole.

(a) Write out the full conditional probability table for $P(s_1|s_0, right_1)$.

$P(s_1\|s_0, right_1)$		s_1				
		0	1	2	3	4
s_0	0	0.1	0.8	0.1	0.0	0.0
	1	0.0	0.1	0.9	0.0	0.0
	2	0.0	0.0	0.4	0.6	0.0
	3	0.0	0.0	0.3	0.1	0.6
	4	0.6	0.0	0.1	0.2	0.1

(b) Let the robot's initial belief be $P(s_0) = (0.4, 0.3, 0.2, 0.1, 0.0)$. Describe the initial belief in English.

The probability that the robot is in a square decreases with the square's distance from the left side of the hallway.

(c) Now compute the robot's new belief after taking a **right** action.

$P(s_1\|s_0, right_1)P(s_0)$		s_1				
		0	1	2	3	4
s_0	0	0.04	0.32	0.04	0.0	0.0
	1	0.0	0.03	0.27	0.0	0.0
	2	0.0	0.0	0.08	0.12	0.0
	3	0.0	0.0	0.03	0.01	0.06
	4	0.0	0.0	0.0	0.0	0.0

Summing out s_0, we get $P(s_1|right_1) = (0.04, 0.35, 0.42, 0.13, 0.06)$

(d) What can I do to check and see that my distribution is reasonable?

Make sure it sums to 1.0.

Observations

(e) Now imagine that the hallway cells have colors. They are colored **blue, red, yellow, red, blue**. The robot is pretty good at observing yellow: on the yellow square it will see yellow with probability 0.9 and either blue or red with probability 0.05. But red and blue are harder. On red and blue squares it will see the correct color with probability 0.7, the other red/blue color with probability 0.25, and yellow with probability 0.05.

What is the full probability distribution over what the robot will see when it looks around after taking one **right** action.

What color is the most likely state? So is the observation distribution just $P(o|2) = (0.05, 0.05, 0.9)$ (red, blue, yellow)?

NO!

$P(o_1\|s_1)$		o_1		
		blue	red	yellow
s_1	0	0.7	0.25	0.05
	1	0.25	0.7	0.05
	2	0.05	0.05	0.9
	3	0.25	0.7	0.05
	4	0.7	0.25	0.05

Now multiply the conditional probability table by the previous belief state.

$P(o_1\|s_1)P(s_1)$		o_1		
		blue	red	yellow
s_1	0	0.028	0.01	0.002
	1	0.087	0.245	0.017
	2	0.021	0.021	0.378
	3	0.033	0.09	0.007
	4	0.042	0.015	0.003

Summing out s_1 gives us $P(o_1) = (0.21, 0.38, 0.41)$ (blue, red, yellow)

(f) Why is the distribution over observations not the same for red and blue, given that the observation probabilities given the state are the same for each color?

(g) What would the belief state be if we saw red, blue or yellow, respectively.

$P(s_1|o_1) = \frac{P(o_1|s_1)P(s_1)}{P(o_1)}$

This is just renormalizing each column from the table above:

$$P(s_1|blue) = \frac{(0.028, 0.087, 0.021, 0.033, 0.042)}{0.21} = (0.13, 0.41, 0.1, 0.16, 0.2)$$

$$P(s_1|red) = \frac{(0.01, 0.245, 0.021, 0.09, 0.015)}{0.38} = (0.02, 0.64, 0.06, 0.24, 0.04)$$

$$P(s_1|yellow) = \frac{(0.002, 0.017, 0.378, 0.007, 0.003)}{0.42} = (0.005, 0.05, 0.92, 0.02, 0.007)$$

(h) Which observation would be the most informative to the robot? Explain your choice.

Yellow. The robot is most sure of its current state if it sees yellow.

(i) The road crew comes and fills in the sinkhole, and while they're at it, they fix the robot's faulty drive system, so it now drive exactly where it wants. That is, a **left** action takes the robot one square left with probability 1.0, and similarly for the **right** action. How does that effect the process we just went through for computing the belief state?

Only the first part of the calculation changes, and it becomes way simpler. You no longer have the write out the whole table.