The three faces of 6.081

- Coping with complexity in software design
- Modeling and interacting with physical systems (control)
- Dealing with error and uncertainty

Organizing view: A framework for abstraction

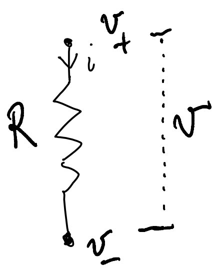
	procedures	data
Primitives	+, *, ==	numbers, strings, True/False
Means of combination	if, while, … composition, e.g., can write 3*(4+7)	data structures: lists dictionaries
Means of abstraction	def	abstract data types classes
Means of capturing common patterns	higher-order procedures	inheritance

Organizing view: linear systems

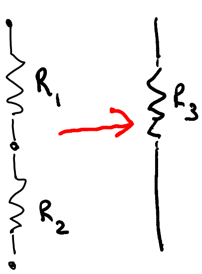
	sequences	systems
primitives	individual sequences	Individual systems
Means of combination	addition scaling shift	cascade parallel sum
Means of abstraction	Z-transform	difference equations system function poles and zeros
Means of capturing common patterns		feedback and Black's formula

Organizing view: circuits

primitives	
Means of combination	
Means of abstraction	
Means of capturing common patterns	

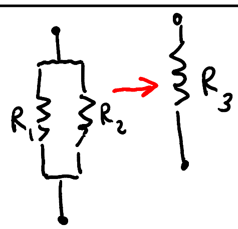


$v = iR$



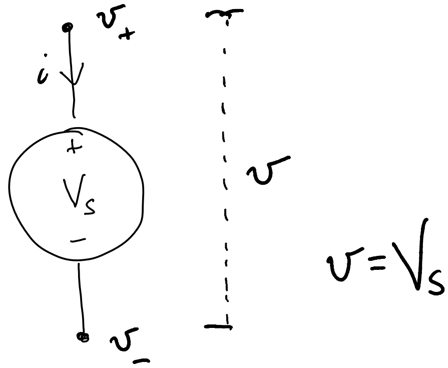
Series combination

$R_3 = R_1 + R_2$

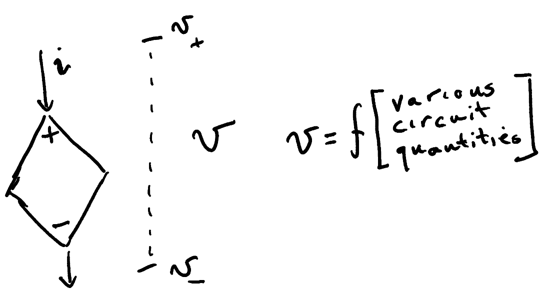


Parallel combination

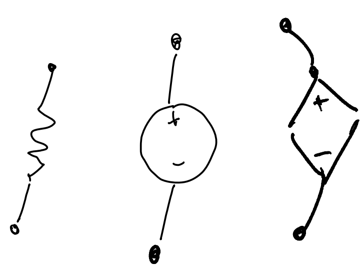
$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2}$$



Ideal independent voltage source



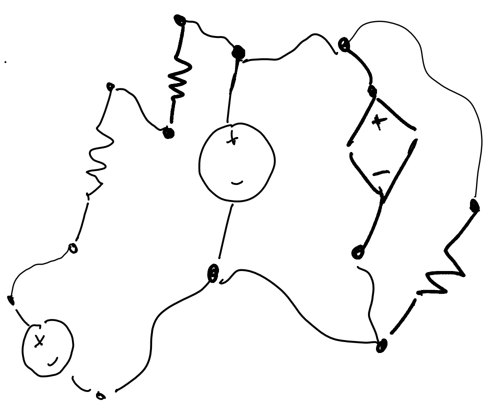
Dependent voltage source



2-terminal devices

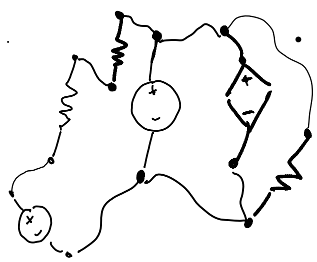
Organizing view: circuits

primitives	resistors, sources, …
Means of combination	??
Means of abstraction	
Means of capturing common patterns	



Organizing view: circuits

primitives	resistors, sources, …
Means of combination	wire things together at nodes
Means of abstraction	
Means of capturing common patterns	

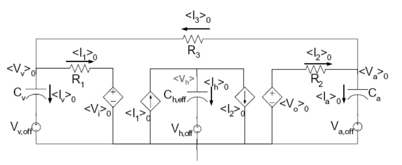


- Nodal analysis
  - Constitutive relations
    - one for each element
  - Conservation Law (Kirchhoff's Current Law)
    - one for each node

"Modeling and Monitoring of Cardiovascular Dynamics in the Intensive Care Unit"
Tushar Parlikar, Thomas Heldt, George Verghese, 2005

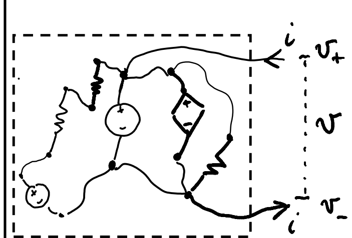
- For the SPCVM, applying the cycle-averaging technique, we obtain a circuit with dependent voltage and current sources:

V_a ~ CVP
V_v ~ ABP
V_h ~ VP



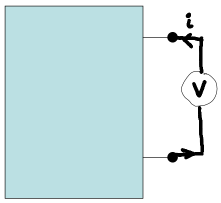
$$\langle V_i(t)\rangle_0 = \langle q_1(t)V_h(t)\rangle_0 + \langle (1-s_1(t))V_1(t)\rangle_0 \approx \langle s_1(t)\rangle_0 \langle V_h(t)\rangle_0 + (1-\langle s_1(t)\rangle_0)\langle V_v(t)\rangle_0 + K_i$$

$$\langle V_o(t)\rangle_0 \approx \langle s_D(t)\rangle_0 \langle V_h(t)\rangle_0 + (1-\langle s_D(t)\rangle_0)\langle V_a(t)\rangle_0 + K_0$$



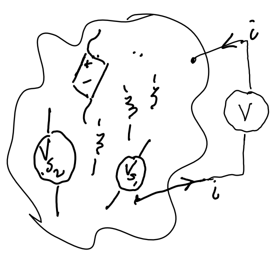
current in = current out

1-port



Apply a voltage and measure the current. The 1-port is completely described by the relation of the between the voltage and the current. **It doesn't matter what's in the box, so long as the relation holds.**

Analogy with software: An abstract data type is described by its API.



in general

$V = V_{TH} + i R_{TH}$

Constitutive relations

$v_1 = i_1 R_1$

$v_2 = V_{S_1}$

$v_3 = \dots$

$v_7 = i_2 + 4 i_3 + V_8 + \dots$

$v = \dots$

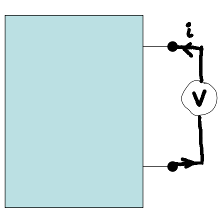
Conservation law

$i_1 + i_2 + i_{11} + \dots = 0$

$i_3 + i_4 + \dots = 0$

$\vdots$

$i + i_7 + i_{10} + \dots = 0$

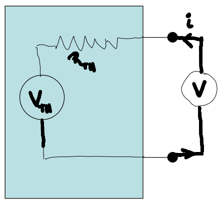


$$v = V_{TH} + iR_{TH}$$

V_{TH} is the voltage when there is an open circuit at the terminals

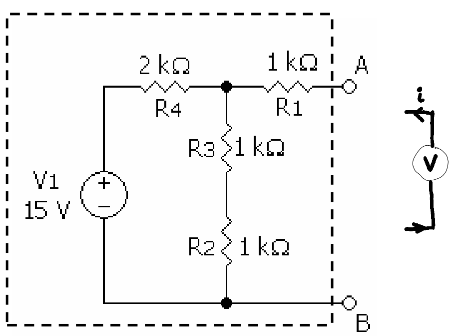
R_{TH} is $v \div i$ when all the independent sources are set to zero

Thévenin's theorem

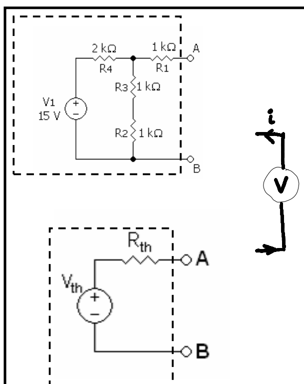


Any two-terminal network made up of resistors and voltage sources, when viewed from the terminals, is **completely electrically equivalent** to a network composed of a single resistor and a single voltage source.

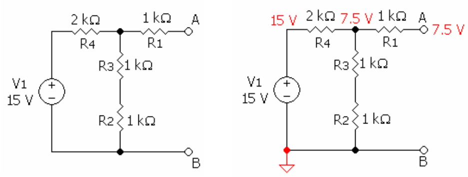
$$v = V_{TH} + iR_{TH}$$



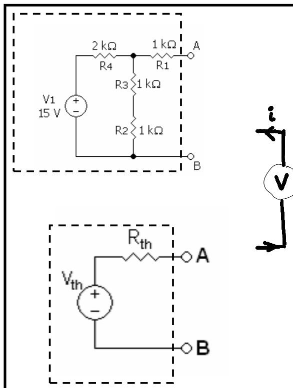
Example: From the Wikipedia



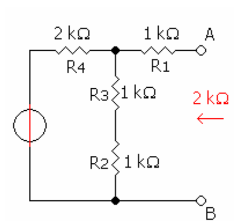
Step 1: The voltage V_{TH} is the voltage we'd see at the terminals if we left them open



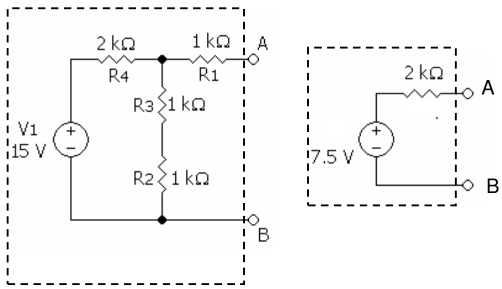
The voltage at the terminals is 7.5V, so V_{TH} is 7.5V



Step 2: The resistance R_{TH} is the resistance we'd see at the terminals when we set the independent source to zero



The resistance seen from the terminals is 2kΩ, so R_{TH} is 2kΩ



Net result: These two circuits are completely electrically equivalent when viewed from the terminals.

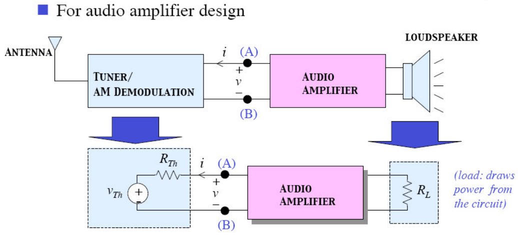
(Analogy: Two different implementations of the same data abstraction.)

Organizing view: circuits

primitives	resistors, sources, …
Means of combination	wire things together at nodes
Means of abstraction	1-port Thévenin equivalent
Means of capturing common patterns	

Thévenin Equivalent

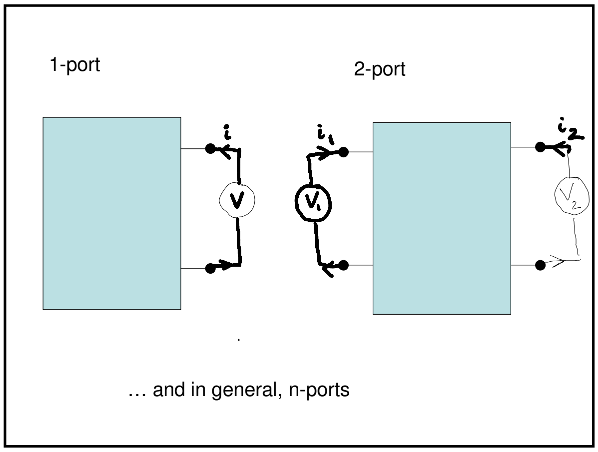
- For audio amplifier design

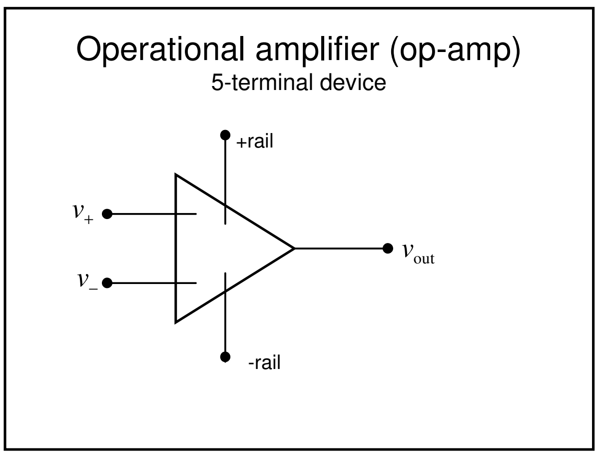


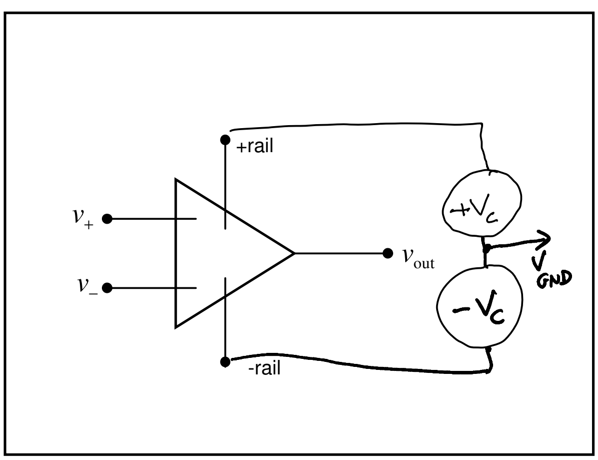
We can **replace** the Tuner/AM Demodulator by a simple **Thevenin Equivalent Circuit** iff the *i*-*v* relationships at terminals A-B are identical.

4

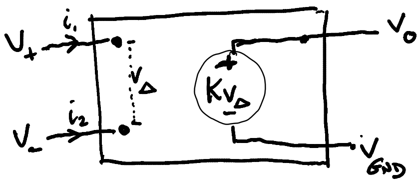
From: Margarida Jacome, UT Austin, EE411


1-port
2-port
i
i1
i2
V
V1
V2
… and in general, n-ports


Operational amplifier (op-amp)
5-terminal device
+rail
v+
v−
vout
-rail


+rail
v+
v−
vout
-rail
+Vc
−Vc
V GND

Ideal op-amp model

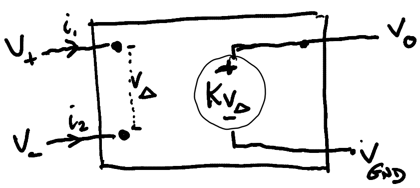


$$v_o - v_{GND} = K(v_+ - v_-)$$

$$i_1 = i_2 = 0$$

K might be around 10,000

Ideal op-amp model



$$v_o - v_{GND} = K(v_+ - v_-)$$

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$$v_o - v_{GND} = K(v_+ - v_-)$$

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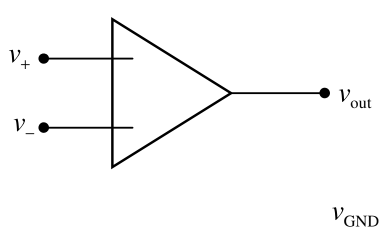
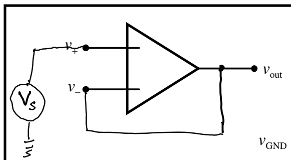




$$v_{\text{out}} = K(v_+ - v_-)$$

$$v_{\text{out}} = K(V_S - v_{\text{out}})$$

$$v_{\text{out}} = KV_S - Kv_{\text{out}}$$

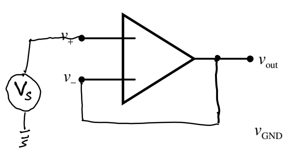
$$v_{\text{out}} + Kv_{\text{out}} = KV_S$$

$$v_{\text{out}}(1+K) = KV_S$$

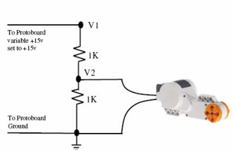
$$\frac{v_{\text{out}}}{V_S} = \frac{K}{1+K}$$

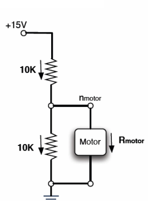
$$\frac{v_{\text{out}}}{V_S} \approx 1$$

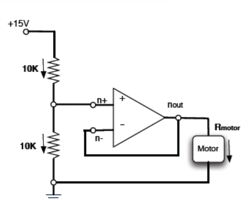
Voltage follower (or buffer)



$$\frac{v_{\text{out}}}{V_S} \approx 1$$



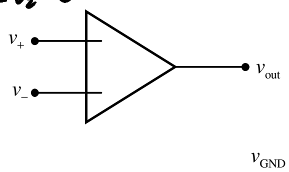




An even simpler op-amp model

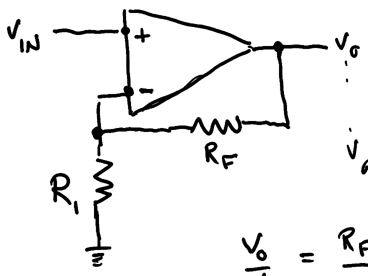
$V_0 - V_{GND} = K(v_+ - v_-)$

$i_1 = i_2 = 0$

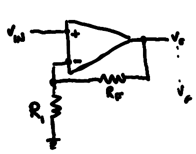


- Draws no current, that is, i1=i2=0
- If K is very large, and v_{out} is finite, then v+=v-

Non-inverting amplifier



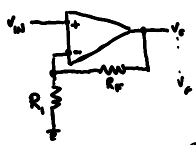
$$\frac{V_O}{V_{IN}} = \frac{R_F + R_1}{R_1}$$

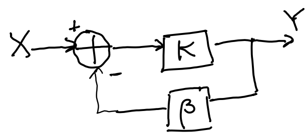


$$\frac{v_o - v_-}{R_F} = \frac{v_-}{R_1}$$

$$v_- = v_+ = v_{in}$$

- $\frac{v_o - v_{in}}{R_F} = \frac{v_{IN}}{R_1}$
- $R_1 v_o - R_1 v_{in} = R_F v_{IN}$
- $R_1 v_o = (R_1 + R_F) v_{IN}$
- $\frac{V_o}{v_{IN}} = \frac{R_F + R_1}{R_1}$

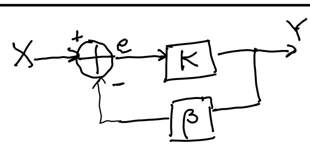




$$\frac{V_o}{v_{IN}} = \frac{R_F + R_1}{R_1}$$

$$\frac{Y}{X} = \frac{K}{1+\beta K} \approx \frac{1}{\beta} \quad K \gg 0$$

$$\beta = \frac{R_1}{R_1 + R_F}$$



$$Y = Ke$$

$$e = X - \beta Y$$

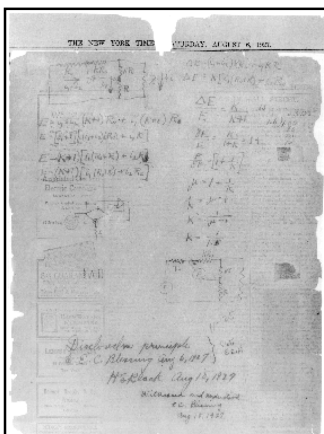
$$Y = K[X - \beta Y]$$

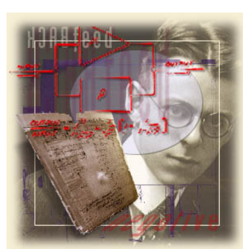
$$Y = KX - \beta KY$$

$$Y[1 + \beta K] = KX$$

$$\frac{Y}{X} = \frac{K}{1+\beta K} \approx \frac{1}{\beta} \text{ if } K \gg 0$$

Black's formula for negative feedback





Harold S. Black (1898-1983)
Inventor of the negative feedback amplifier (1927)