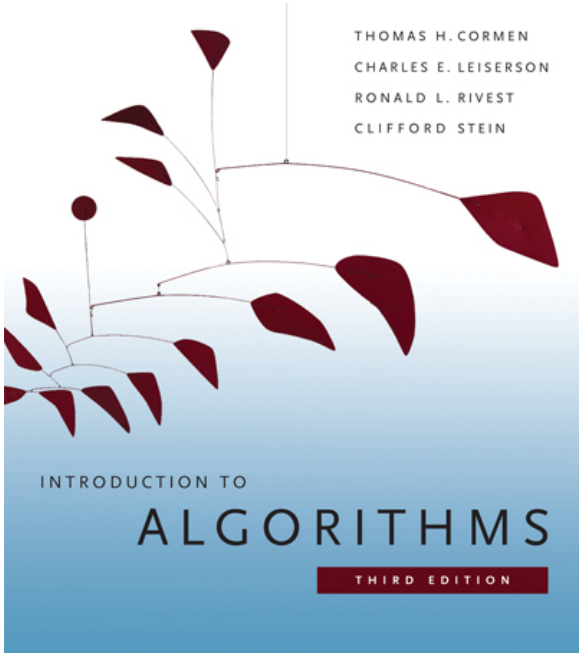
6.006- *Introduction to Algorithms*



Lecture 3

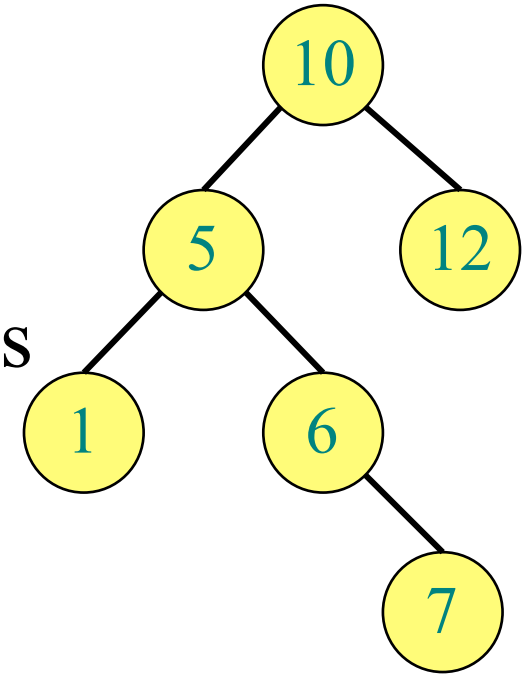
Prof. Costis Daskalakis

Overview

http://izismile.com/tags/Gibraltar/

- Runway reservation system:
  - Definition
  - How to solve with linked-lists
- Binary Search Trees
  - Operations
- Next time: Balanced Search Trees

Readings: CLRS 10, 12.1-3

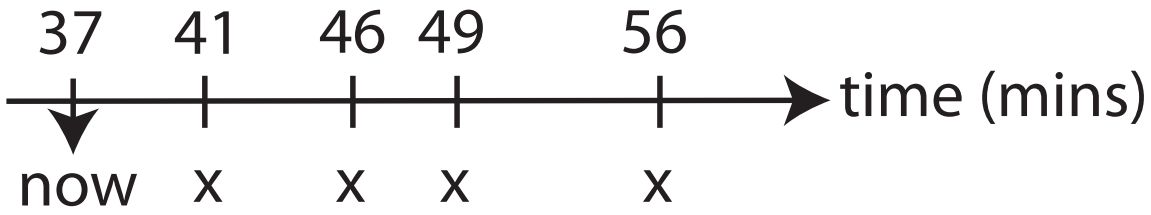


Runway reservation system

- Problem definition:
  - Single (busy) runway
  - Reservations for landings
    - maintain a set of future landing times
    - a new request to land at time t
    - add t to the set if no other landings are scheduled within < 3 minutes from t
    - when a plane lands, removed from the set

Runway reservation system

- Example



  - R = (41, 46, 49, 56)
  - requests for time:
    - 44 => reject (46 in R)
    - 53 => ok
    - 20 => not allowed (already past)
- Ideas for efficient implementation ?

Proposed algorithm

- (keep R as a linked-list)

```
init: R = [ ]

req(t): if t < now: return "error"
for i in range (len(R)):
if abs(t-R[i]) < 3: return "runway busy"
R.append(t)
```

- Complexity?
- Can we do better?

Some other options:

- Keep R as a sorted list:
  - on request **t**, it takes linear time to find the right location in the list where **t** needs to be inserted
  - before inserting **t** at found location check whether the numbers on the left and right of the location are $\leq$**t**-3 and $\geq$**t**+3 respectively

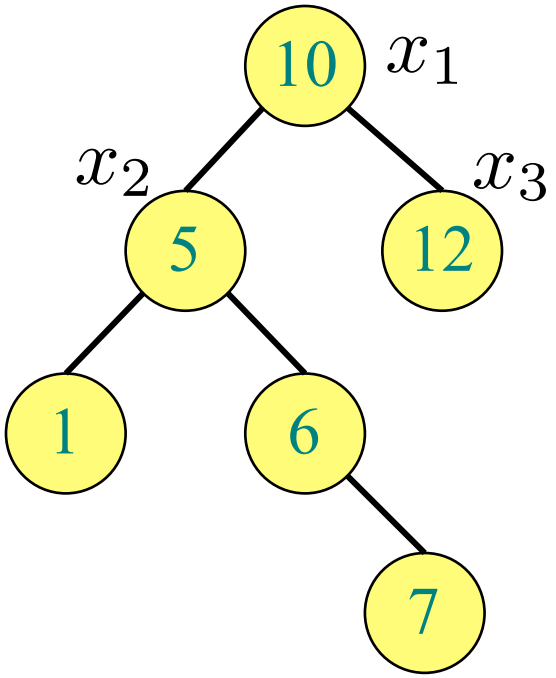
- Keep R as a sorted array:
  - takes O(log n) to find the place to insert new **t**
  - but still requires linear time to actually insert (requires shifting of elements)

Need best of both worlds:

***fast* insertion into *sorted* list**

Binary Search Trees (BSTs)

- A tree …
- …where each node x has:
  - a key[x]
  - three pointers:
    - left[x] : points to left child
    - right[x] : points to right child
    - p[x] : points to parent
- E.g. key[x_1]=10
- left[x_1]=x_2
- p[x_2]=x_1
- p[x_1]=NIL

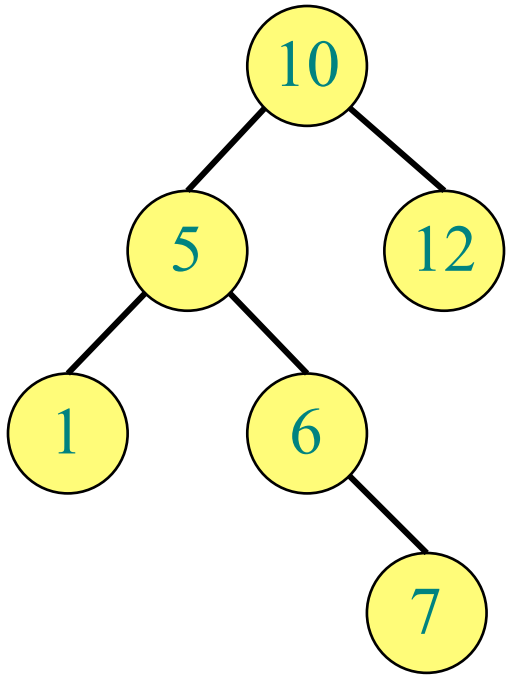


Binary Search Trees (BSTs)

- ***Defining*** property (i.e. what makes it a binary SEARCH tree):
- for any node x:
  - for all nodes y in the left subtree of x:

    $key[y] \leq key[x]$
  - for all nodes y in the right subtree of x:

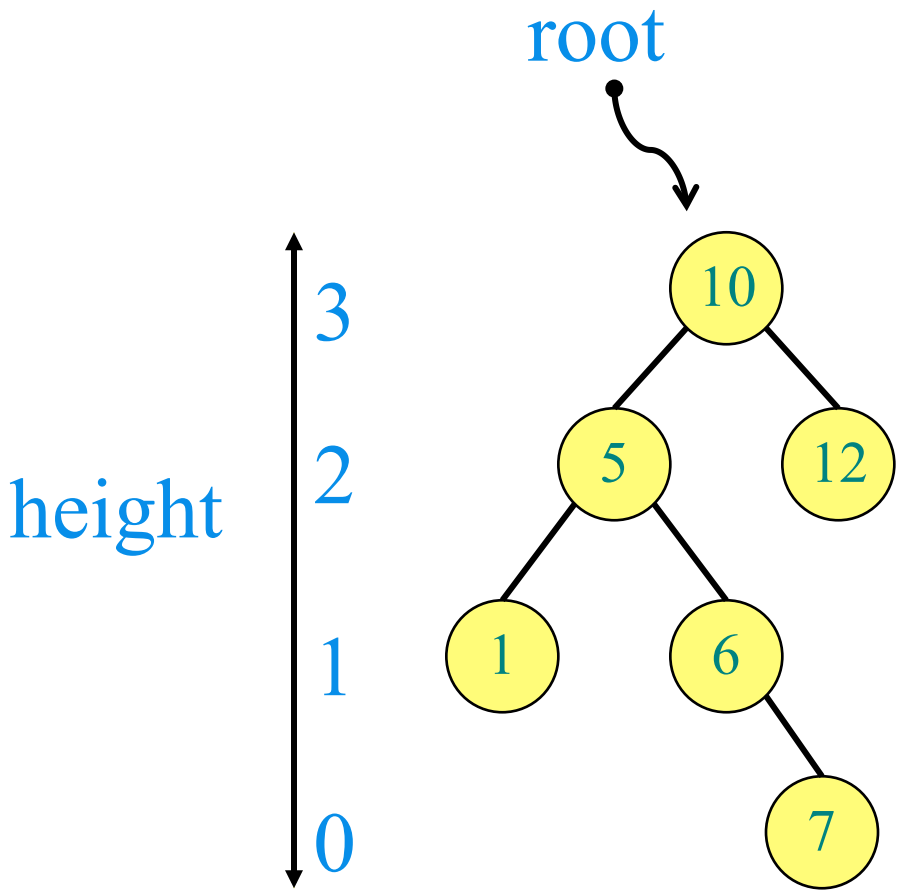
    $key[y] \geq key[x]$



- How are BSTs created?

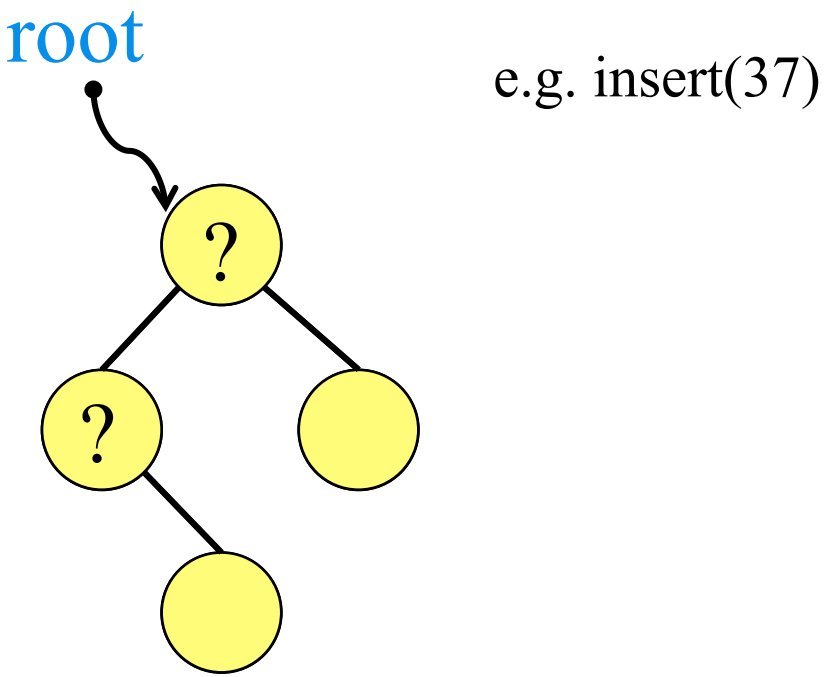
Growing BSTs

- Insert 10
- Insert 12
- Insert 5
- Insert 1
- Insert 6
- Insert 7



BST as a data structure

- Supported Operations:
  - insert(k): insert a node with key k at the appropriate location of the tree



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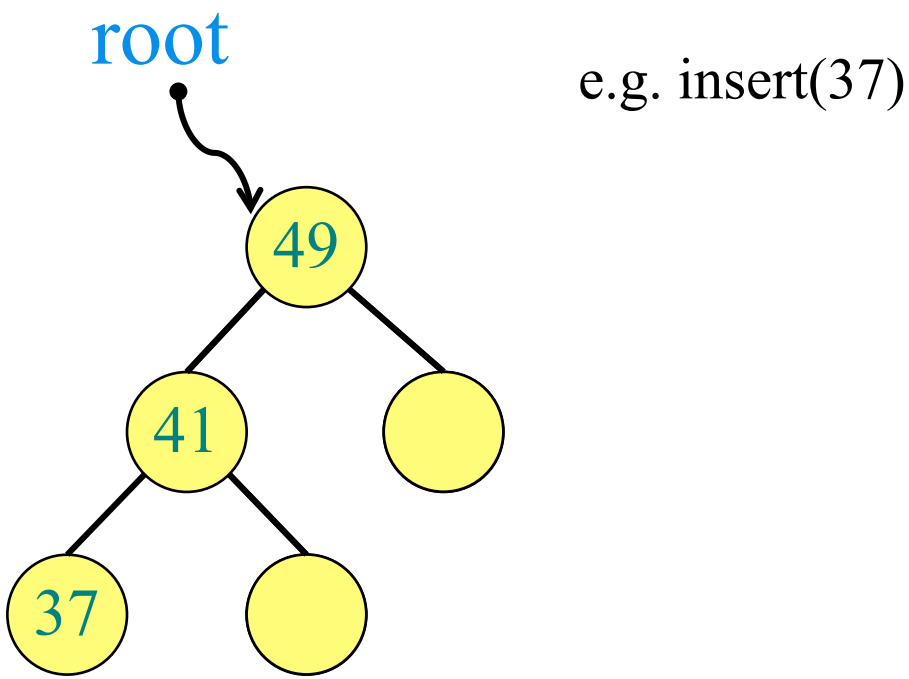
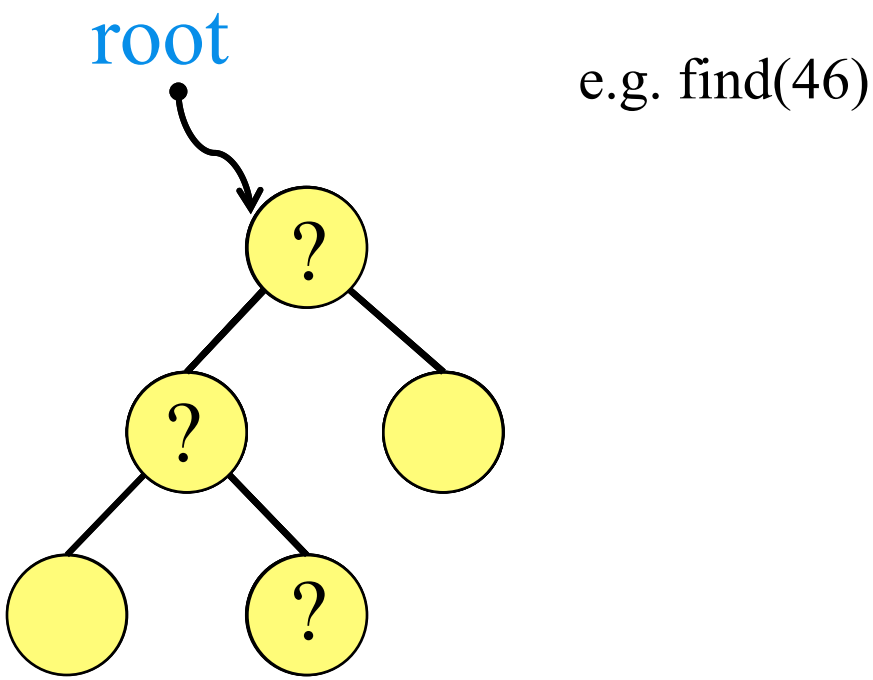


e.g. insert(37)

Aside: Can do the “within 3” check for reservation system during insertion.

BST as a data structure

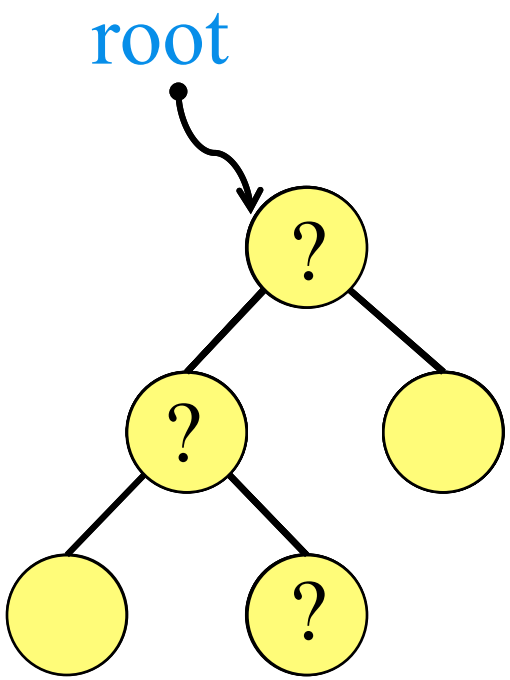
- Supported Operations:
  - find(k): finds the node containing key k (if it exists)



e.g. find(46)

BST as a data structure

- Supported Operations:
  - delete(k): delete the node containing key k, if such a node exists



e.g. delete(46)

BST as a data structure

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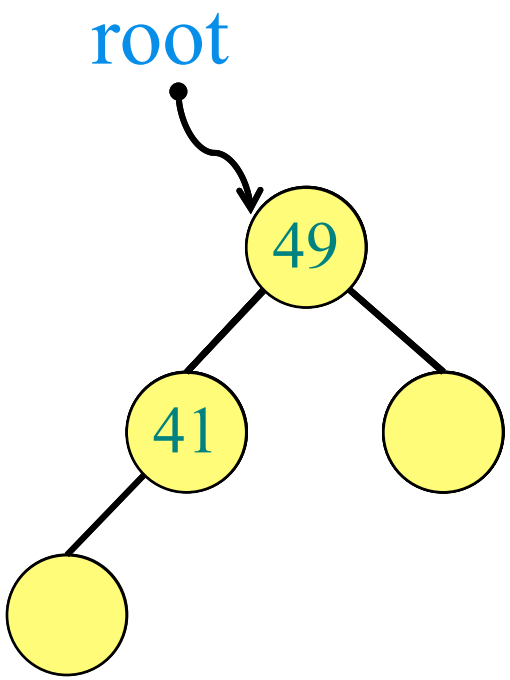


e.g. delete(46)

Question: What if we have to delete a node that is internal? How do we fill in the hole? A: next lecture.

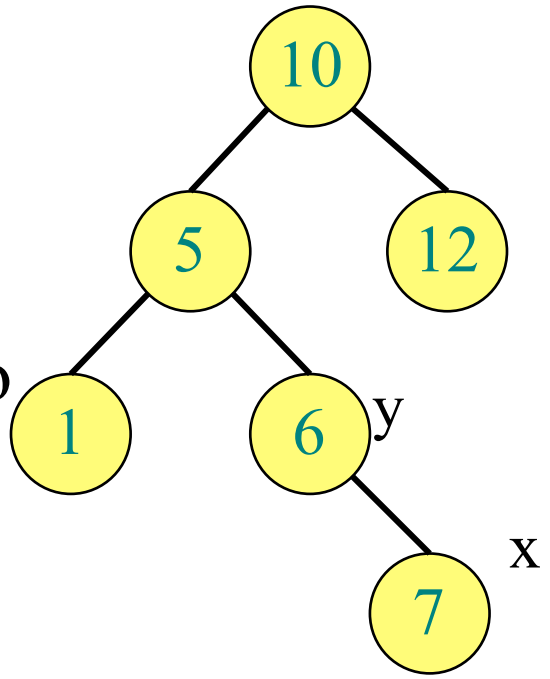
BST as a data structure

- Supported Operations:
  - insert(k): insert a node with key k at the appropriate location of the tree
  - find(k): finds the node containing key k (if it exists)
  - delete(k): delete the node containing key k, if such a node exists
  - findmin(x): finds the minimum of the tree rooted at x
  - deletemin(): finds the minimum of the tree and deletes it
  - next-larger(x): finds the node containing the key that is the immediate next of key[x]

Next-larger

next-larger(x):

- If right[x] ≠ NIL then
  return findmin(right[x])
- Otherwise
  y ← p[x]
  While y≠NIL and x=right[y] do
    - x ← y
    - y ← p[y]
  Return y



next-larger(5) = 6

next-larger(7)

Next-larger

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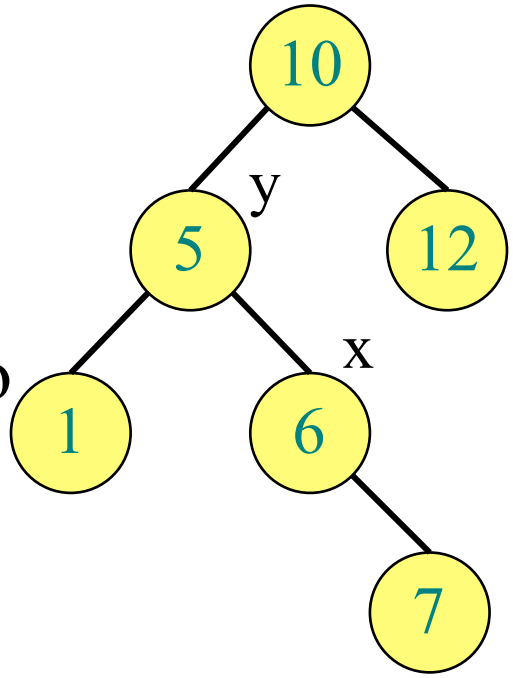

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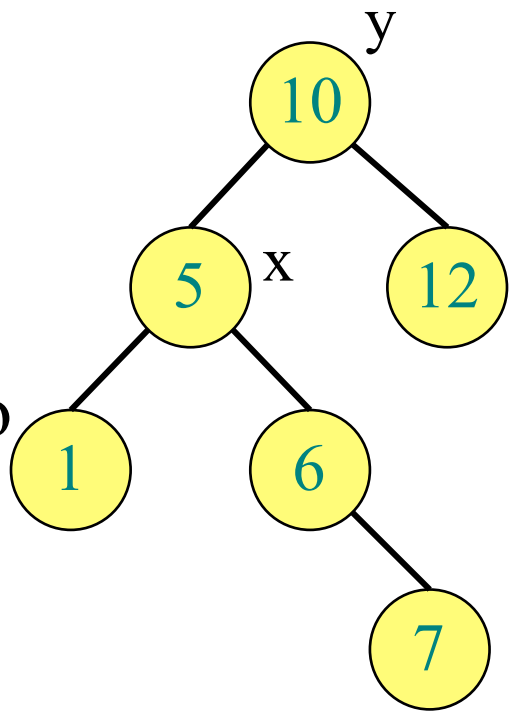


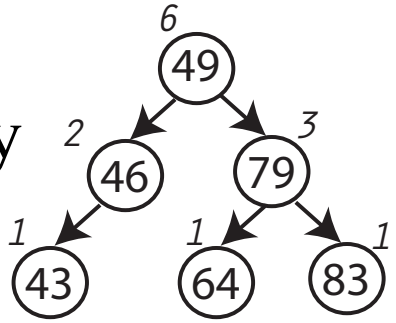
next-larger(5) = 6

next-larger(7) = 10

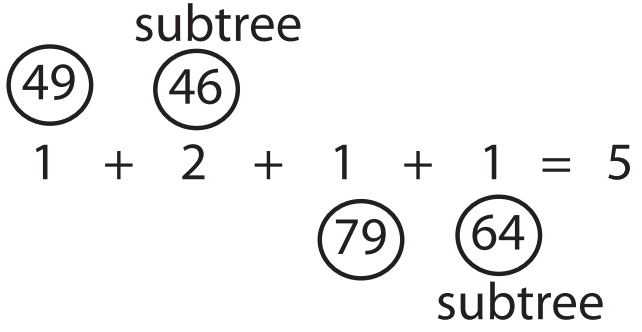
Back to runway reservation system

- Introducing extra requirements: e.g. how many planes are scheduled to land at times ≤ t ?

- Augment the BST structure by keeping track of size of subtrees rooted at all nodes
- To figure out how many planes will land ≤ t:
  - Walk down the tree to find where key t would have been inserted in the tree…
  - … and for every node where you forked to the right:
    - add 1 + size of subtree on the left of that node



e.g. #planes land ≤ 80?

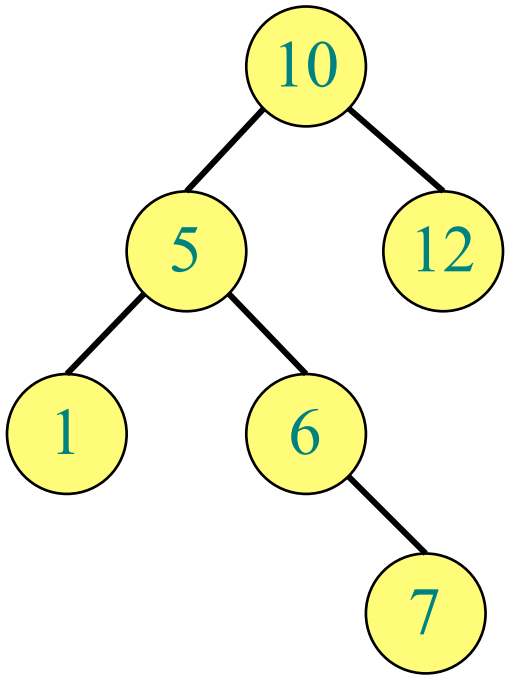


e.g. #planes land ≤ 75? A: 4

Analysis

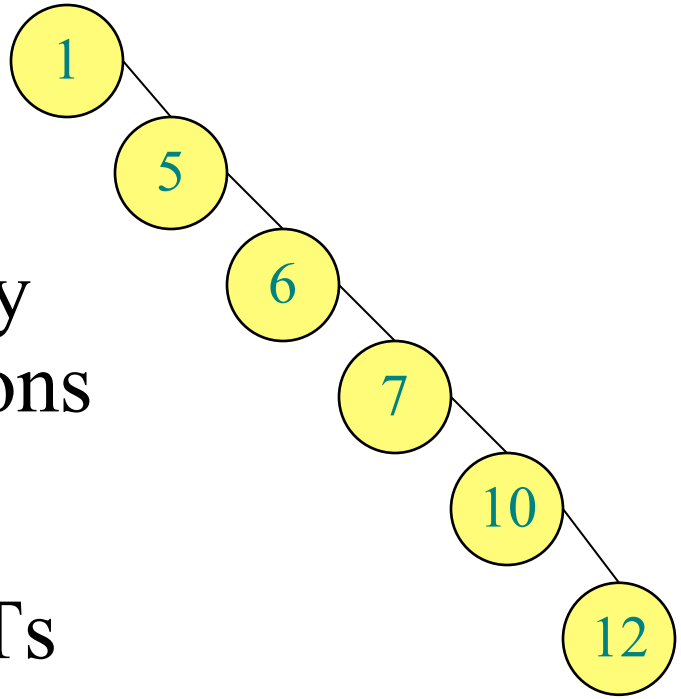
- We have seen insertion, deletion, search, findmin, etc.
- How much time does any of this take ?
- Worst case: O(height)

  => height really important
- After we insert n elements, what is the worst possible BST height ?



Analysis

- n-1



- so, still O(n) for the runway reservation system operations

- Next lecture: balanced BSTs
- **Readings: CLRS 13.1-2**
- **Hw: notice correction in question 4: a '>' was turned to a '≥'**