Final Practice Problems

1 Subset Sum

You are given a sequence of $n$ numbers (positive or negative):

$$x_1, x_2, \ldots, x_n$$

Your job is to select a subset of these numbers of maximum total sum, subject to the constraint that you can’t select two elements that are adjacent (that is, if you pick $x_i$ then you cannot pick either $x_{i-1}$ or $x_{i+1}$).

Explain how you can find, in time polynomial in $n$, the subset of maximum total sum.
2 Collecting Coins

You are given an $n$-by-$n$ grid, where each square $(i, j)$ contains $c(i, j)$ gold coins. Assume that $c(i, j) \geq 0$ for all squares. You must start in the upper-left corner and end in the lower-right corner, and at each step you can only travel one square down or right. When you visit any square, including your starting or ending square, you may collect all of the coins on that square. Give an algorithm to find the maximum number of coins you can collect if you follow the optimal path.
Handout 13: Final Practice Problems

3 True/False

Decide whether these statements are True or False. You must briefly justify all your answers to receive full credit.

1. Any Dynamic Programming algorithm with \( n \) subproblems will run in \( O(n) \) time.
   True  False
   Explain:

2. Karatsuba’s method is based on the use of continued fractions.
   True  False
   Explain:

3. Newton’s Method for computing \( \sqrt{2} \) essentially squares the number of correct digits at each iteration.
   True  False
   Explain:
4 Numerics

Suppose we are trying to compute \( \sqrt[3]{9} \) (the cube root of 9).

Explain carefully how one iteration of Newton’s Method works for this problem, starting with an initial guess of \( x_0 = 2 \). (Hint: the function to use is \( f(x) = x^3 - 9 \).) Be sure to derive carefully the value of \( x_1 \).