6.006	Lecture 5	Feb. 21, 2008

Outline: Hashing I

- Dictionaries & Python
- Motivation
- Hash functions
- Chaining
- Simple uniform hashing
- "Good" hash functions

↳ PS1 due tonight @ 11:59
↳ PS2 out

Reading: CLRS 11.1, 11.2, 11.3

Dictionary problem: Abstract Data Type (ADT)
maintain set of items, each with a key, subject to
- insert(item): add item to set
- delete(item): remove item from set
- search(key): return item with key if it exists

- assume items have distinct keys (or that inserting new one clobbers old)
- balanced BSTs solve in $O(\lg n)$ time per op. (in addition to inexact searches like nextlargest)
- **goal**: $O(1)$ time per operation

Python dictionaries: items are (key, value) pairs

e.g. `d = {'algorithms': 5, 'cool': 42}`

```
d.items()    → [('algorithms', 5), ('cool', 5)]
d['cool']    → 42
d[42]        → KeyError
'cool' in d  → True
42 in d      → False
```

- Python **set** is really **dict** where items are keys

Motivation: Document Distance

- already used in

```
def count_frequency(word_list):
    D = {}
    for word in word_list:
        if word in D:
            D[word] += 1
        else:
            D[word] = 1
```

- new docdist7 uses dictionaries instead of sorting:

```
def inner_product(D1, D2):
    sum = 0.0
    for key in D1:
        if key in D2:
            sum += D1[key] * D2[key]
```

$\Rightarrow$ optimal $\Theta(n)$ document distance
assuming dictionary ops. take $O(1)$ time

Motivation: PS2

How close is chimp DNA to human DNA?

= Longest common substring of two strings

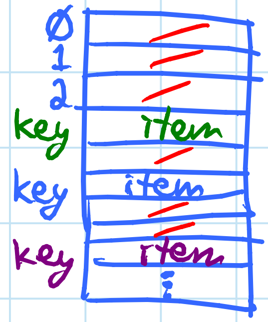
e.g. ALGORITHM vs. ARITHMETIC

- dictionaries help speed algorithms

e.g. put all substrings into set, looking for duplicates $\Big\} \Theta(n^2)$

How do we solve the dictionary problem?

Simple approach: Direct-access table



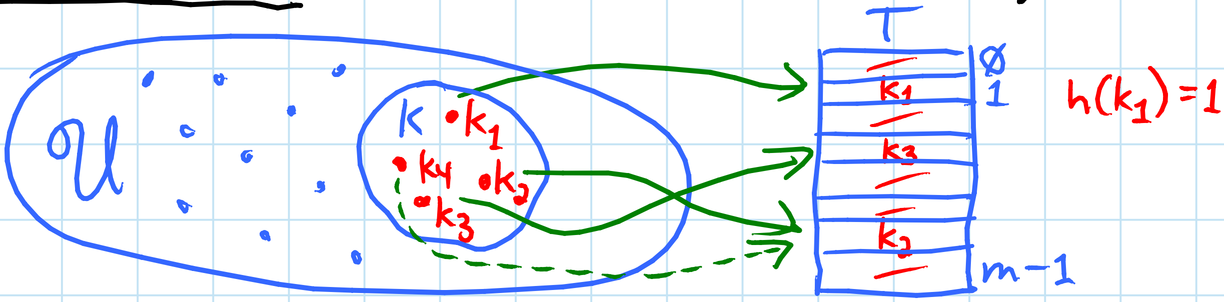
- store items in an array, indexed by key
- problems:
  ① keys must be nonnegative integers
  (or, using two arrays, integers)
  ② large key range ⇒ large space
  e.g. one key of 2^{256} is bad news

Solution ①: map key space to integers

- in Python: hash(object) where object is a number, string, tuple, etc., or object implementing __hash__
  misnomer: should be called "prehash"
- ideally, $x = y \iff hash(x) = hash(y)$
- Python applies some heuristics
  e.g. hash('\ØB') = 64 = hash('\Ø\ØC')
- object's key should not change while in table
  (else can't find it anymore)
  - no mutable objects like lists

Solution ②: hashing (verb from 'hache' = hatchet, Germanic)

- reduce universe $\mathcal{U}$ of all keys (say, integers) down to reasonable size m for table
- idea: $m \approx n$, $n = |K|$, K = keys in dictionary
- hash function $h: \mathcal{U} \to \{0, 1, \ldots, m-1\}$

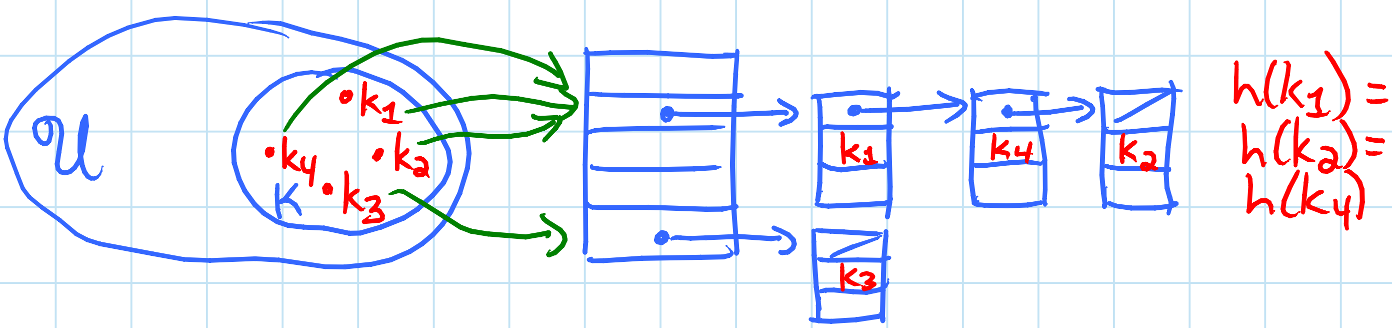


- two keys $k_i, k_j \in K$ collide if $h(k_i) = h(k_j)$

How do we deal with collisions? we'll see two ways

- chaining: TODAY
- open addressing: NEXT WEEK

Chaining: linked list of colliding elements in each slot of table



- search must go through whole list $T[h(key)]$
- worst case: all keys in K hash to same slot
  $\Rightarrow \Theta(n)$ per operation

Simple uniform hashing: an **assumption:**
each key is equally likely to be hashed to any slot of table, independent of where other keys are hashed

- let n = # keys stored in table
  m = # slots in table
- **load factor** $\alpha = n/m$
  = average # keys per slot

Expected performance of chaining: assuming

$O(1+\alpha)$

search the list

apply hash function & access slot

[actually $\Theta(1+\alpha)$, even for successful search; see CLRS]

$= O(1)$ if $\alpha = O(1)$ i.e. $m = \Omega(n)$

Hash functions:

Division method: $h(k) = k \bmod m$

- k_1 & k_2 collide when $k_1 \equiv k_2 \pmod m$
  i.e. when m divides $|k_1 - k_2|$
- fine if keys you store are uniform random
- but if keys are $x, 2x, 3x, \ldots$ (regularity)
  and x & m have common divisor d
  then use only $1/d$ of table
  - likely if m has a small divisor e.g. 2
- if $m = 2^r$ then only look at r bits of key.

- GOOD PRACTICE: m is a prime
  & not close to power of 2 or 10
  (to avoid common regularities in keys)
- inconvenient to find prime; division slow

Multiplication method: $h(k) = [(a \cdot k) \bmod 2^w] \gg (w-r)$

where $m = 2^r$ & w-bit machine words
& a = odd integer between 2^{w-1} & 2^w

- GOOD PRACTICE: a not too close to 2^{w-1} or 2^w
- faster: multiplication & bit extraction