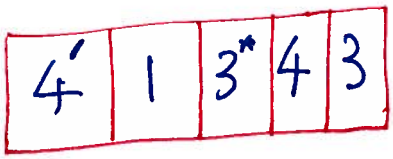
Sorting IV

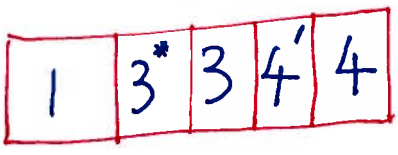
6.006
SPRING 2008
L11

Stable sorting
Radix Sort
Quick sort ← ~~6.006~~
Sorting races

Stable Sorting

Preserves input order among equal elements





counting sort is stable
mergesort is stable

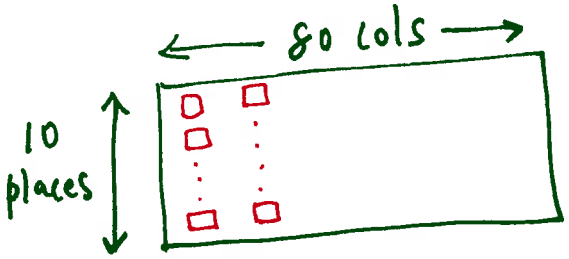
Selection sort : Find maximum element and put it at end of array
& (swap with element at end of array)
Heap

not stable! $3\ 2_a\ 2_b \rightarrow 2_b\ 2_a\ 3$

define $2_a < 2_b$

Radix Sort

- Herman Hollerith card-sorting machine for 1890 census
- Digit by Digit sort by mechanical machine



1) examine given column of each card in a deck
2) Distribute the card into one of 10 bins
3) Gather cards bin by bin, so cards with first place punched are on top of cards with second place punched, etc.

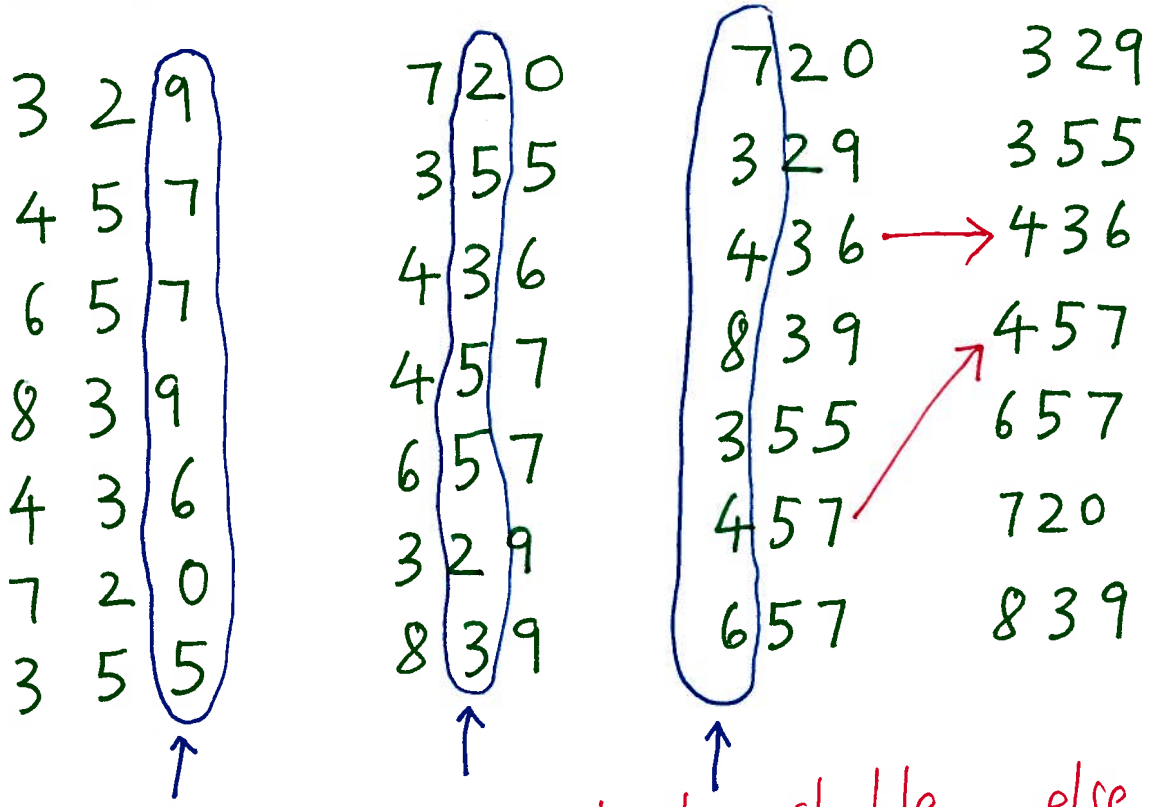
MSB vs LSB ?

Sort on most significant digit first or least significant digit first ?

MSB strategy: Cards in 9 of 10 bins must be put aside, leading to a large number of intermediate piles

LSB strategy: Can gather sorted cards in bins appropriately to create a deck!

EXAMPLE



Digit sort needs to be stable, else will get wrong result!

Analysis

Assume counting sort is auxiliary stable sort. $\Theta(n+k)$ complexity

Suppose we have n words of b bits each.

One pass of counting sort $\Theta(n+2^b)$

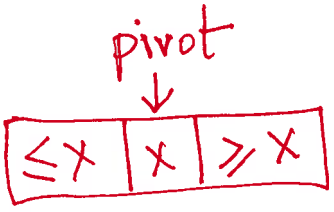
b passes of counting sort $\Theta(b(n+2)) = \Theta(nb)$

$\frac{b}{r}$ passes $\Theta\left(\frac{b}{r}(n+2^r)\right)$

minimized when $r = \lg n$ $\Theta\left(\frac{bn}{\lg n}\right)$

QUICKSORT "enrichment"

Divide: partition the array into two subarrays around a pivot x such that elements in lower subarray $\leq x \leq$ elements in upper subarray. ← Linear time



Conquer: Recursively sort the two subarrays

Combine: Trivial.

If we can choose a pivot such that two subarrays are roughly equal

$$T(n) = 2T(n/2) + \theta(n)$$
$$\Rightarrow T(n) = \theta(n \lg n)$$

If one array is much bigger

$$T(n) = T(n-1) + \theta(n)$$
$$\Rightarrow T(n) = \theta(n^2)$$

Average case $\theta(n \lg n)$ assuming input array is randomized!

Sorting Races

http://cg.scs.carleton.ca/~morin/misc/sortalg

Bubblesort: $\Theta(n^2)$

Repeatedly step thru list to be sorted. Compare 2 items, swap if they are in the wrong order. Continue thru list, until no swaps. Repeat pass thru list until no swaps

Shellsort: $\Theta(n \log^2 n)$

Improves insertion sort by comparing elements separated by gaps

Bubble sort vs. Insertion sort
last element put in place — each element put in right place

Heap sort vs. Merge sort vs. QuickSort