6.006 Recitation

Build 2008.25

6.006 Proudly Presents

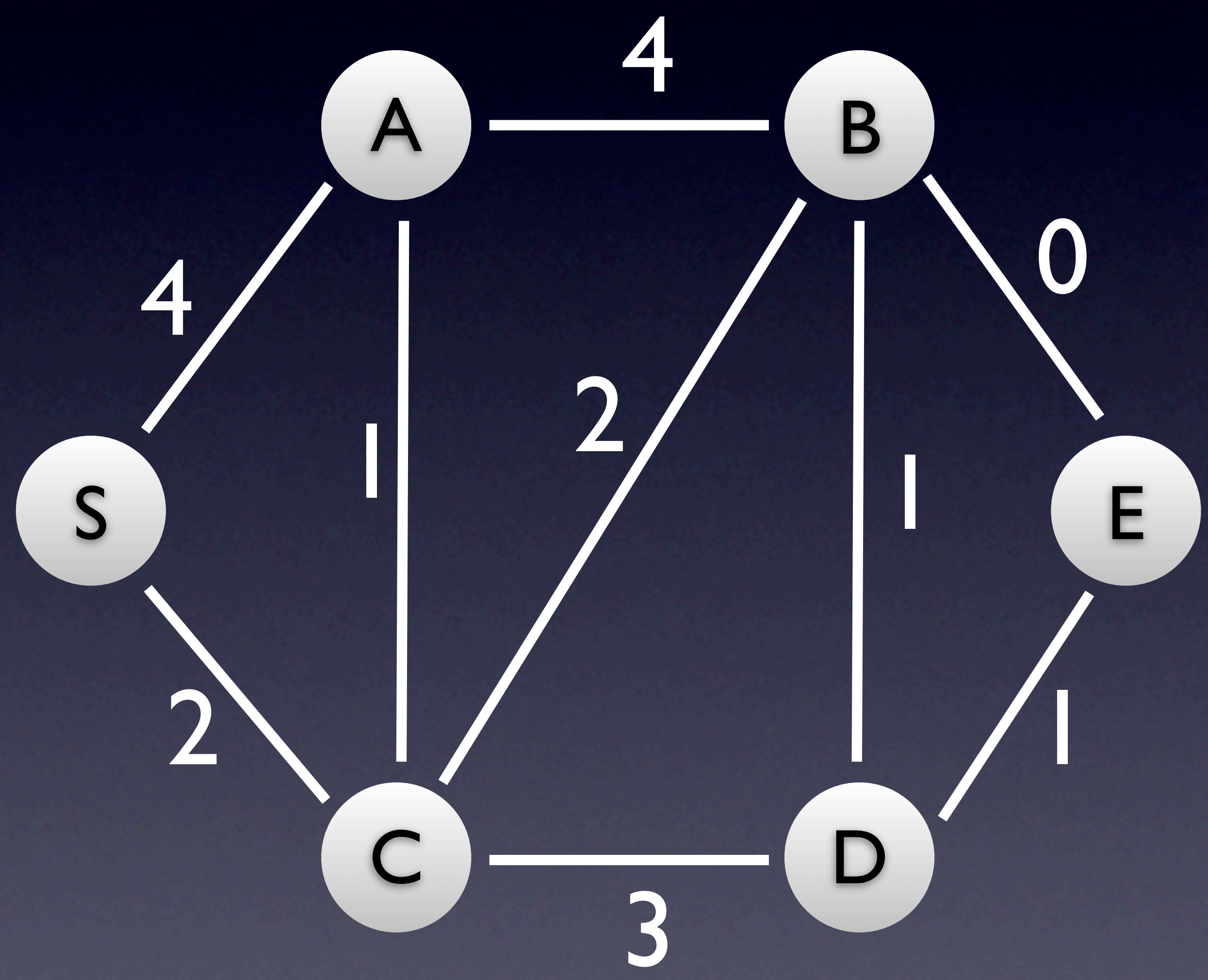
- Dijkstra: minimum-cost paths on crack
  - Algorithm
  - Concepts
  - Implementation
- Data structures come back from the dead (not talking about the quiz)

Minimum-Path Problem

- Given: graph G, source vertex s, edge costs
- Want: paths from s to everything else with minimum costs (sum of edge costs)
- Approach: let d[v] be upper bounds for the real minimum costs, δ[v]
- Start out easy: d[v] = ∞, d[s] = 0
- Relax until values in d converge to δ

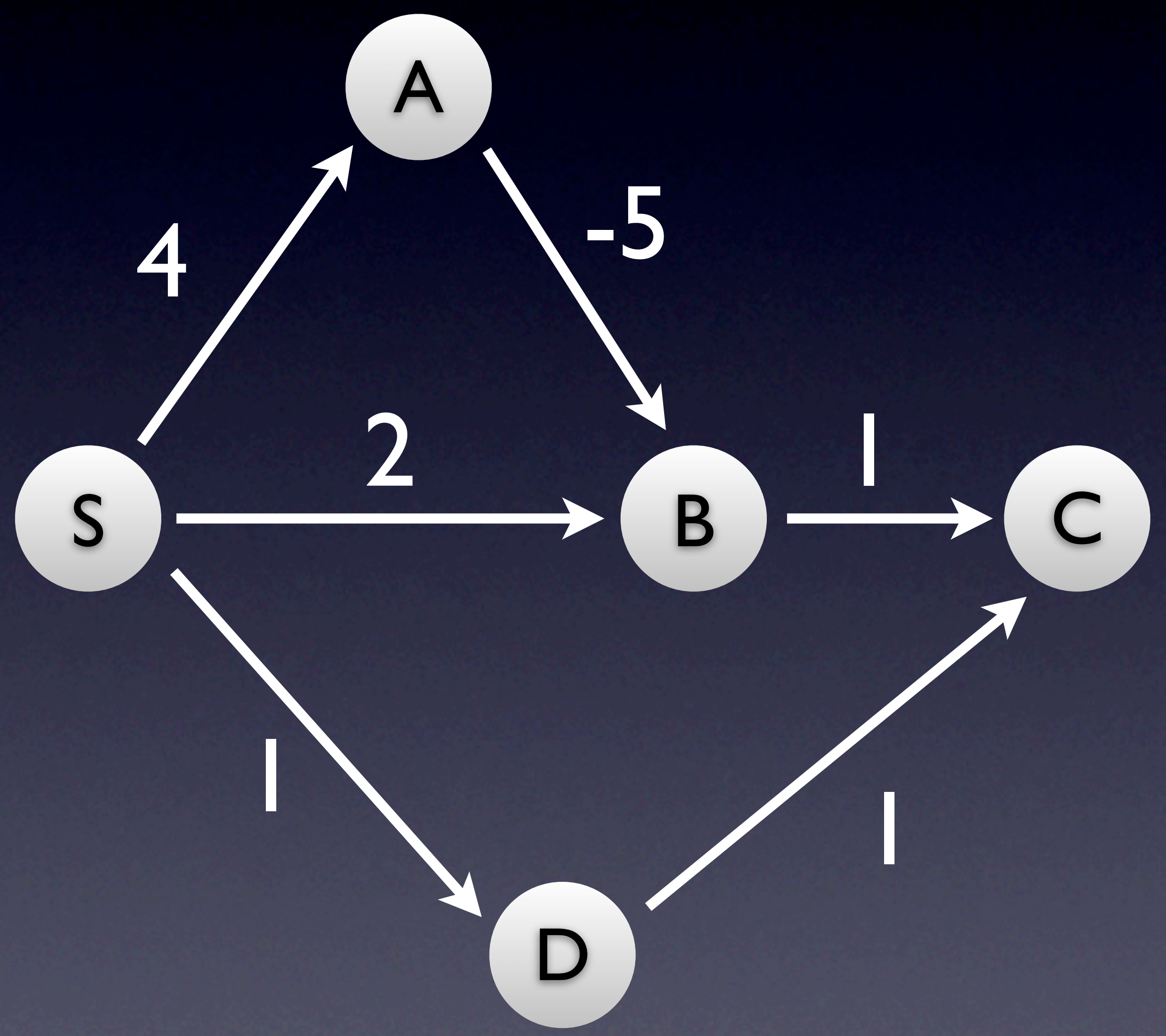
Good Dijkstra

- Generic initialization
- U = V
- Choose v = argmin d[v in U], remove v from U
- Notice d[v] = δ[v]
- Relax v's outgoing edges
- Rinse, repeat



Bad Dijkstra

- Generic initialization
- U = V
- Choose v = argmin d[v in U], remove v from U
- Notice d[v] = δ[v]
- Relax v's outgoing edges
- Rinse, repeat



Dijkstra Overview

- Nice and fast (that's why it's on crack)
- With limitations (crack impacts judgement)
  - Doesn't handle negative-cost edges
    - DOES handle 0-cost edges
  - Harder to code than Bellman-Ford

Dijkstra Works: Intuition

Dijkstra Works: Formal

Making Dijkstra Fast (its crack dealer)

- Generic initialization: $d[v] \leftarrow \infty$, $d[S] = 0$
- Choose $v = \text{argmin}\ d[v]$, by now $d[v] = \delta[v]$
- Relax all edges going out of v
- Rinse, repeat

- Computing argmin
  - V times
- Relaxing
  - E times
- Looks like we need a Data Structure

Min-Priority Queues

- Data Structure
  - **insert**(key) : adds to the queue
  - **min**() : returns the minimum key
  - **delete-min**() : deletes the min key
  - **delete**(key) : deletes the given key
    - optional (only needed in some apps)

Priority Queues with Min-Heaps

- Costs (see above line for explanations)
  - insert: O(log(N))
  - min: O(1)
  - delete-min: O(log(N))
  - delete: O(log(N)) - only if given the index of the node containing the key

Priority Queues with PS3

- Is this priority queue monotone?
- Profit

Cool Python: Generators

1. Iterators
   - used in for loops
   - objects implementing next()
2. Generators
   - express iterator functionality in a cooler way

```
def counter():
    i = 0
    while True:
        yield i
        i += 1


----
c = counter()
c.next()
>> 0
c.next()
>> 1
d = counter()
d.next()
>> 0
c.next()
>> 2
d.next()
>> 1
c.next()
>> 3
```

Dijkstra-Ready Priority Queues

```python
class heap_id:
    def __init__(self):
        self.A = [None]
        self.heapsize = 0
        self.ID_to_index = {}
        self._ID = self._ID_generator()
   def insert(self, key):
       """Returns an ID that is associated with the item."""
       self.heapsize += 1
       ID = self._ID.next()
       self.ID_to_index[ID] = self.heapsize
       self.A.append( [positive_infinity(), ID] )
       self.decrease_key(self.heapsize, key)
       return ID
   def _ID_generator(self):
       ID = 0
       while True:
           yield ID
           ID += 1
```

Dijkstra-Ready Priority Queues II

```python
class heap_id:
    def decrease_key_using_id(self, ID, key):
        """Decrease key given ID."""
        self.decrease_key(self.ID_to_index[ID], key)
    def extract_min(self):
        """Extracts min and returns key."""
        return self.extract_min_with_id()[0]
    def extract_min_with_id(self):
        """Extracts min and returns (key,ID) pair."""
        if self.heapsize < 1:
            print "error: empty heap"
            return
        self._swap(1, self.heapsize)
        self.heapsize -= 1
        min_pair = self.A.pop()
        del self.ID_to_index[min_pair[1]]
        self.min_heapify(1)
        return tuple(min_pair)
```

And we’re done!

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v. Next

- Drawings to help Dijkstra intuition, and make it go faster (or not fail completely)
- Consider eliminating intuition if the formal method is taught
- The Dijkstra run was boring, add spice to it (don't remove, we needed to go through both runs to clarify negative cycle issues)