http://courses.csail.mit.edu/6.006

Administrivia

Course overview

"Peak finding" problem
- 1D version
- 2D version

Course Overview

- Efficient procedures for solving problems on large inputs (e.g., US highway map, human genome)
- Scalability
- Classic data structures and elementary algorithms (CLRS text)
- Real implementations in Python
- Fun problem sets

Content

8 modules each with motivating problem and problem set(s) (except last)

Algorithmic thinking : Peak finding
Sorting & Trees : Event simulation
Hashing : Genome comparison
Numerics : RSA encryption
Graphs : Rubik's cube
Shortest Paths : Caltech → MIT
Dynamic Programming : Image compression
Advanced Topics

TENTATIVE

PEAK FINDER

One-dimensional version

1	2	3	4	5	6	7	8	9
a	b	c	d	e	f	g	h	i

a-i are numbers

Position 2 is a peak if and only if
$b \geq a$ and $b \geq c$

Position 9 is a peak if $i \geq h$

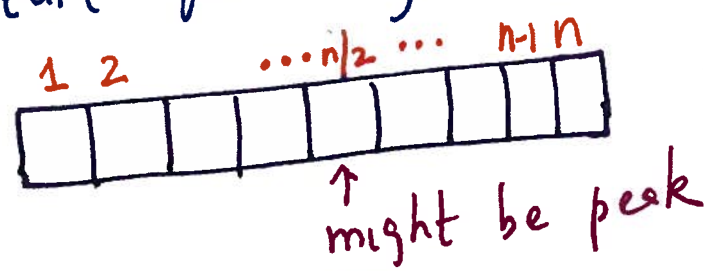
Problem: Find a peak if it exists.*

* Does it always exist?

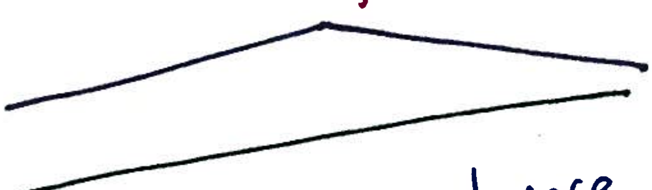
6	7	4	3	2	1	4	5

STRAIGHTFORWARD ALGORITHM

Start from left

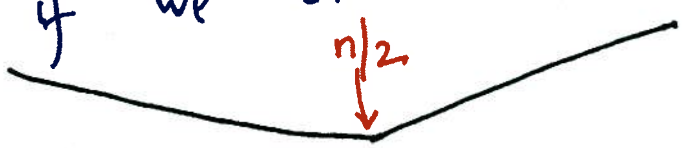


Look at n/2 elements
Could look at n elements



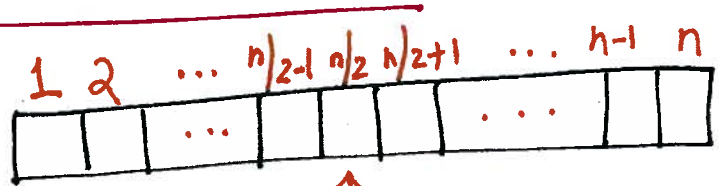
$\Theta(n)$ complexity worst case

What if we start in the middle?



Look at n/2 elements

Can we do better?



Divide & Conquer

Look at n/2 position

If $a[n/2] < a[n/2-1]$ then only look at left half $1 \,..\, n/2-1$ to look for peak

Else if $a[n/2] < a[n/2+1]$ then only look at right half $n/2+1 \,..\, n$ to look for peak

Else n/2 position is a peak

WHY? $a[n/2] \geq a[n/2-1]$

$a[n/2] \geq a[n/2+1]$

What is the complexity?

$$T(n) = T(n/2) + \Theta(1)$$

To compare* $a[n/2]$ to neighbors

$$= \Theta(1) + \dots + \Theta(1) \quad (\log_2 n \text{ times})$$

$$T(n) = \Theta(\log_2 n)$$

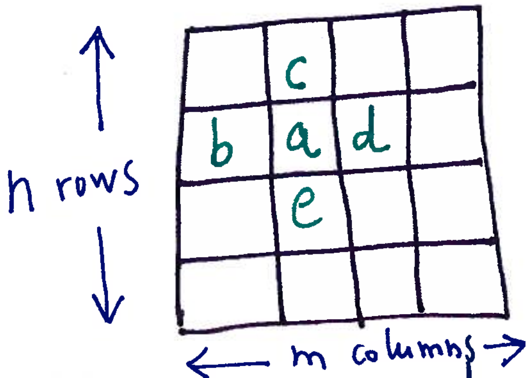
$n = 1{,}000{,}000$ $\Theta(n)$ algo 13 s in python impl

$\Theta(\log n)$ algo 0.001 s

Argue that the algorithm is correct

* In order to sum up the $\Theta(1)$'s as we do here, we need to find a constant that works for all.

2-Dimensional Version

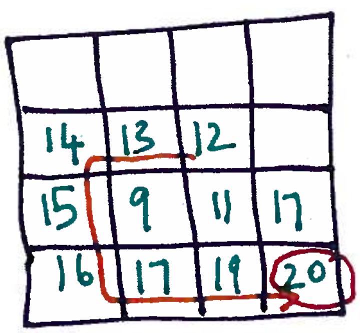


a is 2D peak iff

$a \geq b$, $a \geq d$, $a \geq c$, $a \geq e$

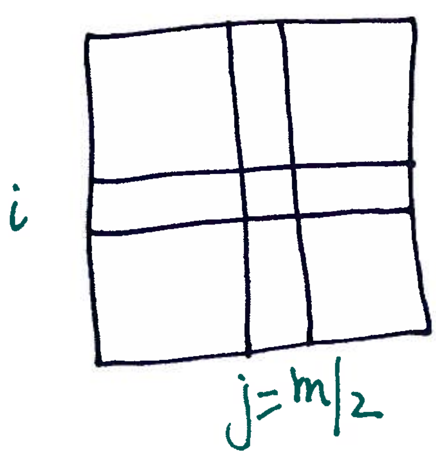
Greedy ascent algorithm: $\Theta(nm)$ complexity

$\Theta(n^2)$ algorithm if $m = n$



○ peak

Extend 1D divide & conquer to 2D: Attempt #1



Pick middle column $j = m/2$

Find a 1D peak at i, j

Use (i,j) as a start point on row i to find 1D-peak on row i

ATTEMPT #1 FAILS

Problem: 2D peak may not exist on row i

		10	
14	13	12	
15	9	11	
16	17	19	20

end up with 14 which is not a 2D peak

ATTEMPT # 2

Pick middle column $j = m/2$

Find global maximum on column j at (i, j)

Compare $(i, j-1)$, (i, j), $(i, j+1)$

Pick left cols if $(i, j-1) > (i, j)$

(similarly for right)

(i, j) is a 2D-peak if neither condition holds ←

Solve the new problem with half the number of columns

When you have a single column, find global maximum and you're done

EXAMPLE OF ATTEMPT #2

10	8	10	10
14	13	12	11
15	9	11	21
16	17	19	20

↑ pick this column

17 global maximum for column

go with

10	10
12	11
11	21
19	20

↑ pick this column

19 is global maximum for column

10
11
21
20

find 21

COMPLEXITY OF ATTEMPT #2

n rows, m columns

$$T(n, m) = T(n, m/2) + \Theta(n)$$

$\Theta(n)$ ↓ to find global maximum on a column (n rows)

$$T(n,m) = \underbrace{\Theta(n) + \dots \Theta(n)}_{\log m} = \Theta(n \log m)$$

$$= \Theta(n \log n) \quad \text{if } m = n$$

Q: What if we replaced global maximum with 1D-peak in Attempt #2? Would that work?