6.006 RECITATION 4 13 Feb 2009

- Binary Search Trees
- Lecture 4
- AVL Trees

AVL TREES

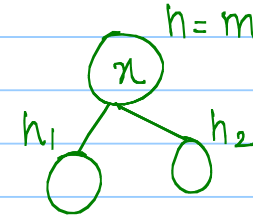
G.M. Adelson-Velsky and E.M. Landis
"An algorithm for the organization of information", 1962

Balanced Binary Search Tree

For any node x, height(x) : Length of longest path from x to a leaf node

$h = \max(h_1, h_2) + 1$

$\text{height}(x) = \max(\text{height}(\text{left}(x)), \text{height}(\text{right}(x))) + 1$



$\text{height}(\text{NULL}) = -1$, $\text{height}(y) = 0 \ \forall y$: leaf node

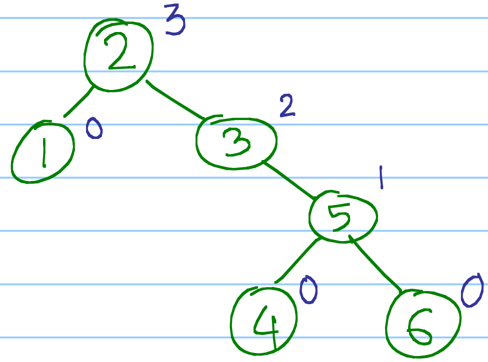
AVL invariant

$\forall x$: node, $|\text{height}(\text{left}(x)) - \text{height}(\text{right}(x))| \leq 1$ $\qquad |h_1 - h_2| \leq 1$

$\text{height}(\text{root}) < 2 \log n$

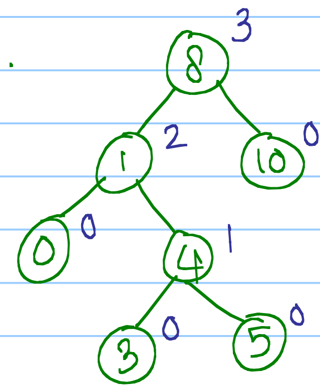
EXCERCISE

A.



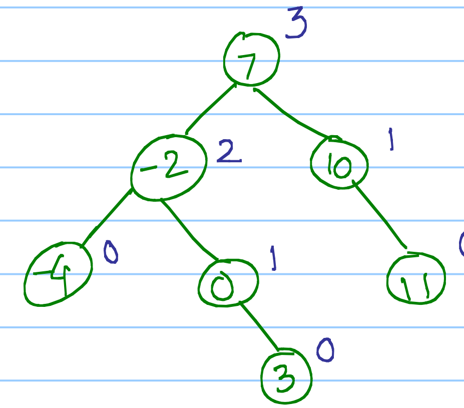
AVL ✗

B.



AVL ✗

C.



AVL ✓

$$
\begin{aligned}
N_h &\geq N_{h-1} + N_{h-2} + 1 \\
&\geq 2N_{h-2} + 1 \\
&\geq 1 + 2(1 + 2N_{h-4}) \\
&\geq 1 + 2 + 2^2(1 + 2N_{h-6}) \\
&\geq 1 + 2 + 2^2 + \ldots\ldots\ 2^{h/2} = 2^{h/2+1} - 1
\end{aligned}
$$

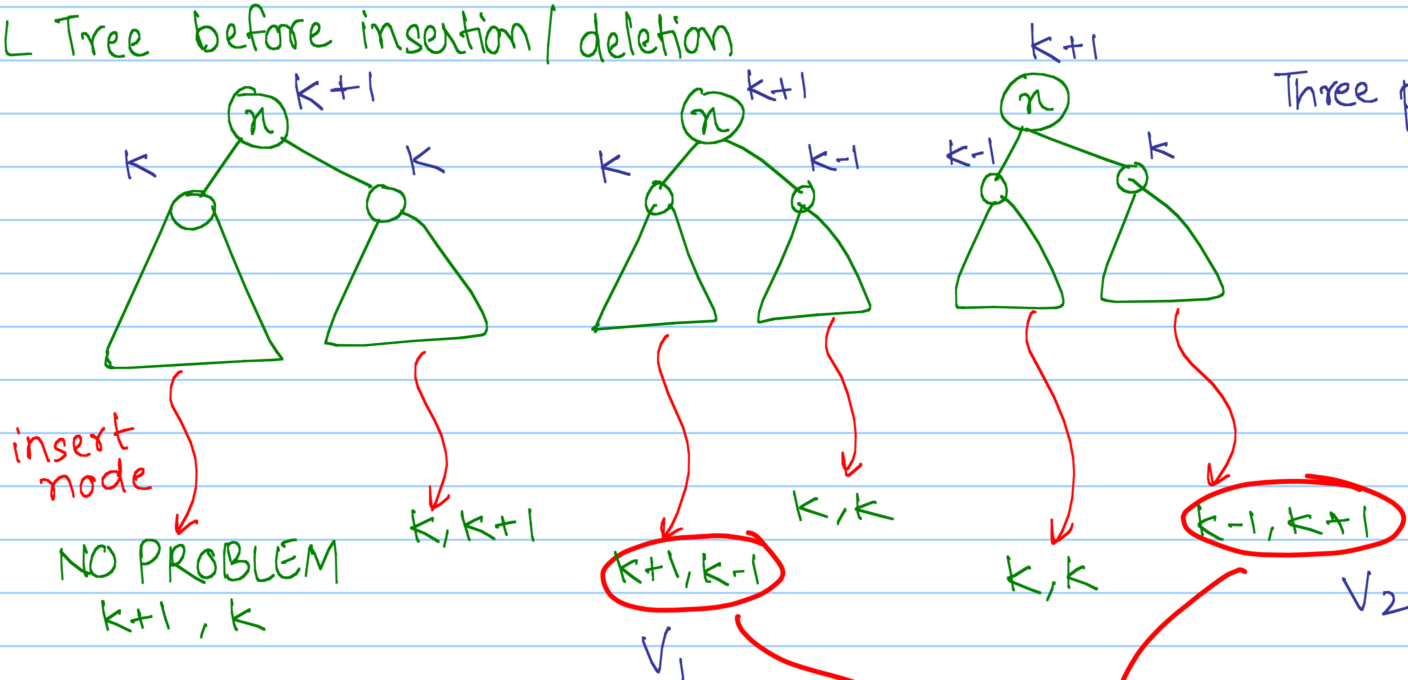
$$N_h \geq 2^{h/2}$$

$$h \leq 2 \log N_h$$

ROTATIONS

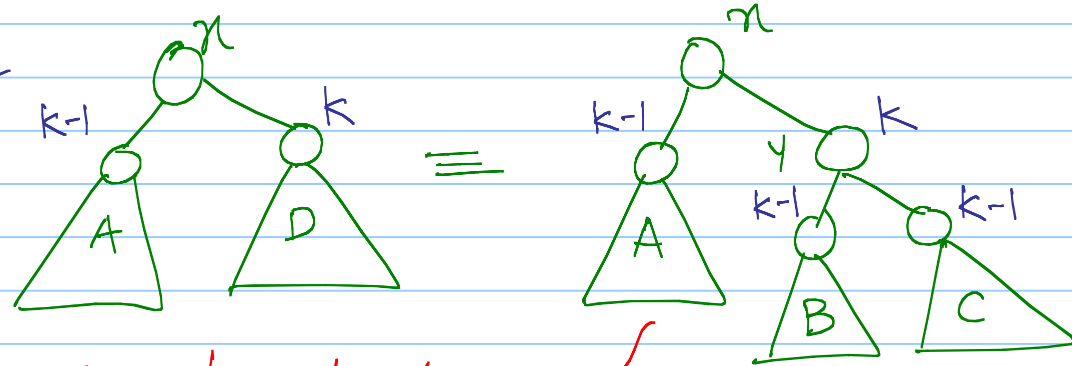
AVL Tree before insertion / deletion

Three possible cases



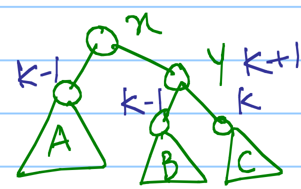
AVL invariant violated !

Consider a violation V_2

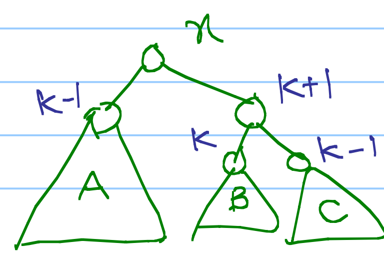


insert node α

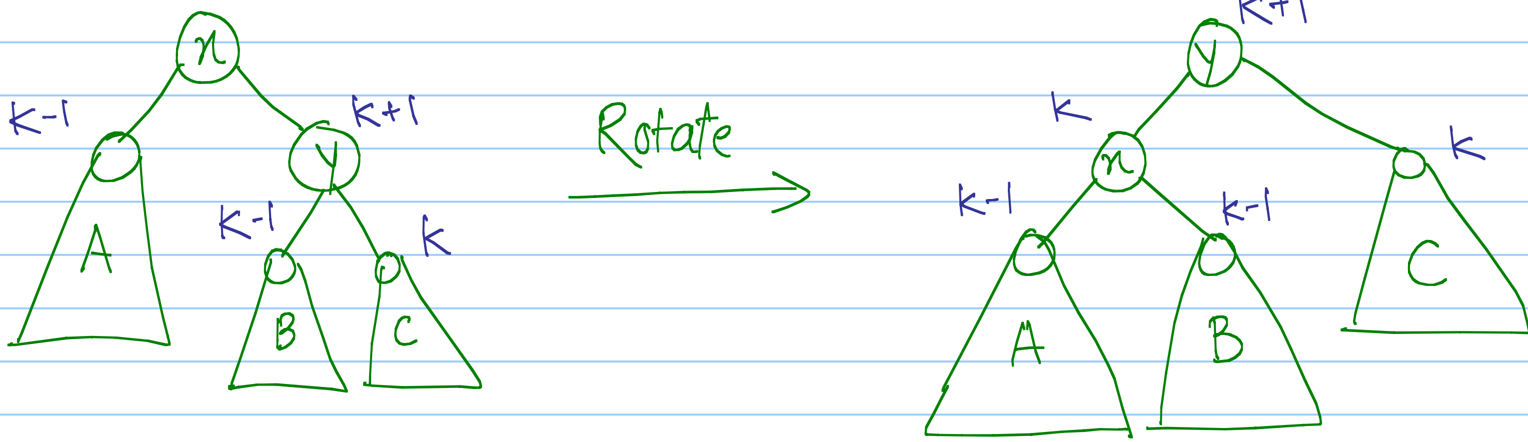
Case 1 : goes to C



case 2 : goes to B



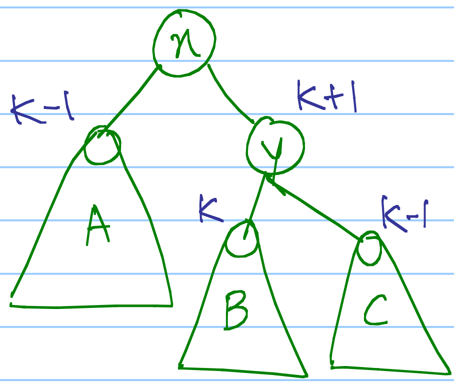
Case 1:

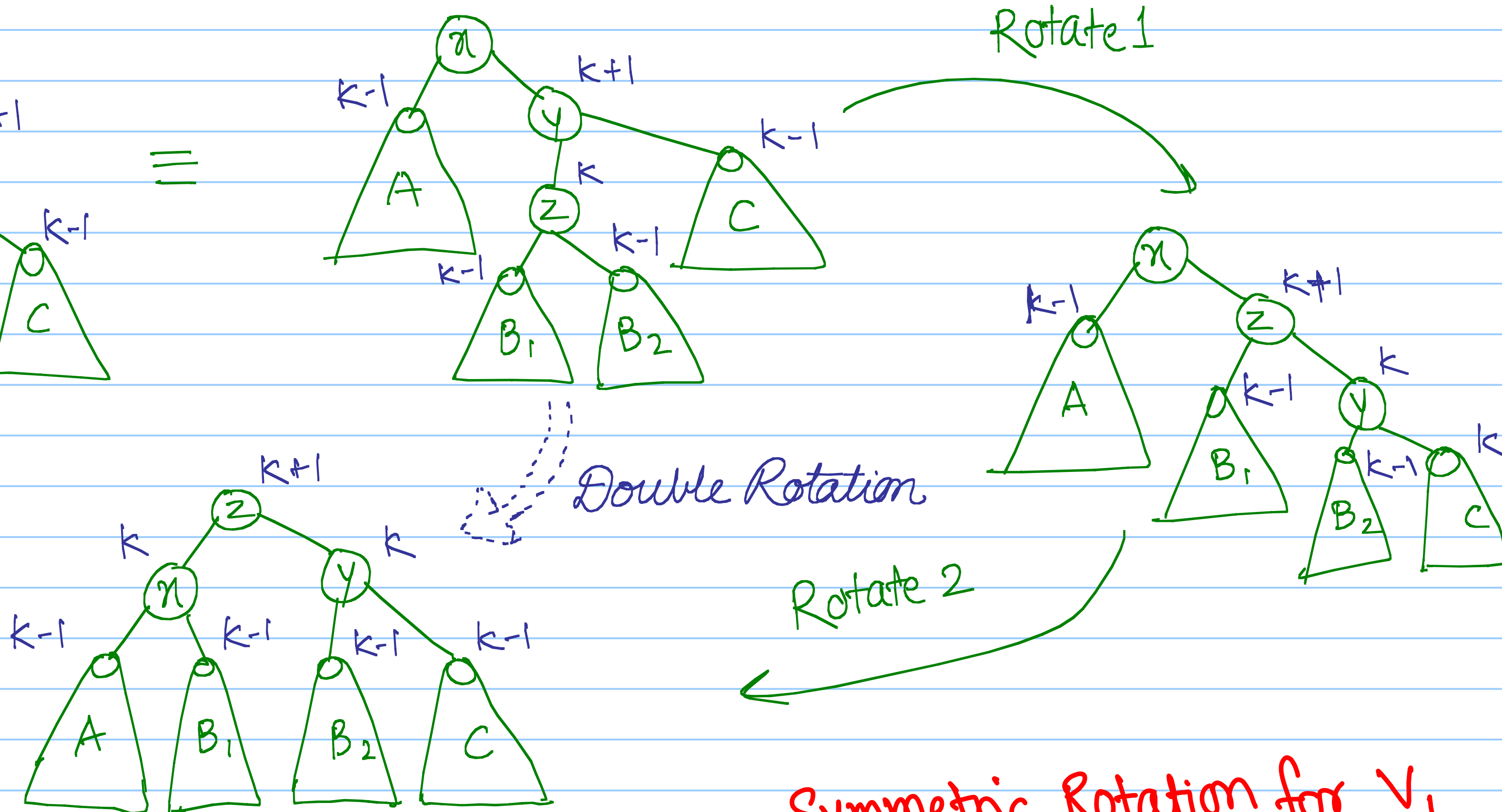


AVL :)

```
def left_rot(x):
        y = x.right
        x.right = y.left
        y.left = x
    return y
```

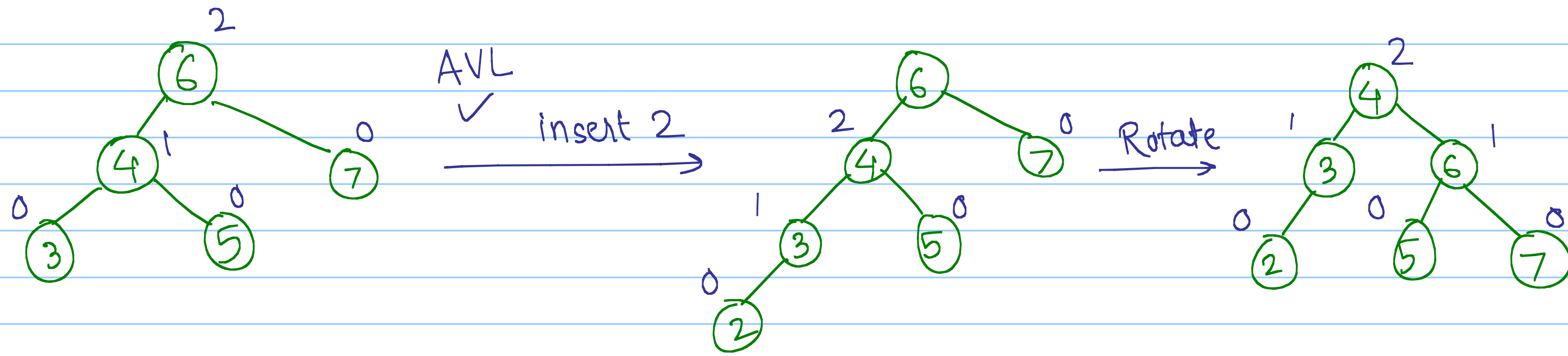
Case 2:






```
def dbl_left_rot(x):
    y = x.right
    z = y.left
    x.right = z.left
    z.left = x
    y.left = z.right
    z.right = y
    return z
```

Symmetric Rotation for V_1



INSERTION AND DELETION

Similar as in BST, followed by rotations to correct imbalances

* INSERTION — one rotation is sufficient (why?) height of root of subtree remains k+1 after rotation

* DELETION — need to check AVL invariant from first point of discrepancy to the root.

O(log n)

Deletion From AVL Trees Excercise