6.006
Rivest
L12.1
10/14/2008

Admin:

- Quiz: Wed. night 7:30-9:30 in 1-190
  (conflict: Th 8am & 1pm - see TA's)
  bring one sheet of notes
- No recitations tomorrow
- Psets back (end of class)

Readings: CLRS: 22.1-22.3, B.4

Outline: Search (1 of 3)

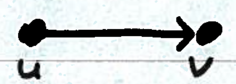
- ☐ graphs review & apps
- ☐ graph representations
- ☐ intro to BFS (breadth-first search) & DFS (depth-first search)

Graph search: to explore a graph.
to find a path from a start vertex s to a desired target vertex

Recall: graph $G = (V, E)$

where V = (finite) set of vertices
E = set of edges (vertex pairs) $\subseteq V \times V$

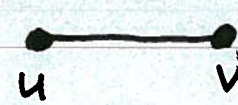
if G is directed graph, then edge is ordered pair (u, v)
(i.e. from u to v)



"can get from u to v"

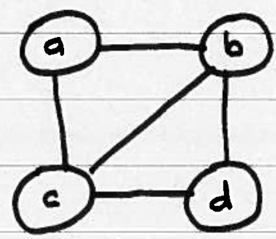
if G is undirected graph, then edge is unordered pair $\{u, v\}$
representing connection between u & v both ways

"can go both ways"



Examples

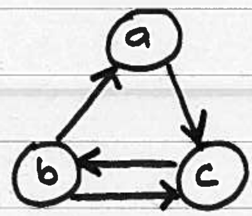
Undirected graph



$V = \{a, b, c, d\}$

$E = \{\{a,b\}, \{a,c\}, \{b,c\}, \{b,d\}, \{c,d\}\}$

Directed graph



$V = \{a, b, c\}$

$E = \{(a,c), (b,c), (c,b), (b,a)\}$

Note: self-loops & multiple edges typically excluded
often write (u,v) for edge even if G is undirected
(understand that it means unordered pair, though)

Graph has <u>two</u> parameters describing its size:

$|V|$ = # of vertices

$|E|$ = # of edges $\quad (0 \leq |E| < |V|^2)$

Running times may be described as function of both.

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Applications: many !!!

- web crawling (how Google finds pages)
- internet routing
- solving puzzles & games

Pocket Cube (2x2x2 Rubik's cube)

(next pset!)

- defines configuration graph:
  - vertex for each possible state
  - edge for each basic move (e.g. 90° turn) (180°?? too) from one state to another
- puzzle: given some initial state s, find path to solved state
- $|V| = 8! \cdot 3^8 = 264{,}539{,}520$

  ← rotate cubelet

  8 cubelets in arbitrary positions

- This is big! Can we simplify?
- Factor out 24-fold symmetry: fix position & orientation of one cubelet

  $|V| = 7! \cdot 3^7 = 11{,}022{,}480$

- In fact, graph as 3 "connected components"

  

  only 1/3 of states are reachable (without dissembling cube)

  Only need to search one component

  $|V| = 7! \cdot 3^6 = 3{,}674{,}160$

Breadth-first search of configuration graph

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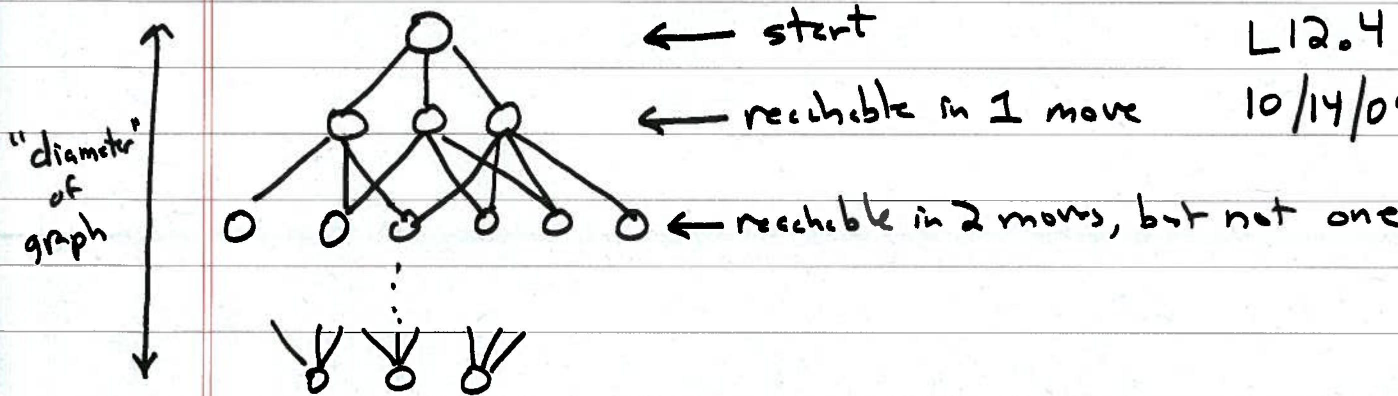

distance	# reachable configs: 90° turns	# reachable configs: 90° & 180° turns
0	1	1
1	6	9
2	27	54
3	120	321
4	534	1,847
5	2256	9,992
6	8,969	50,136
7	33,058	227,536
8	114,149	870,072
9	360,508	1,887,748
10	930,588	623,800
11	1,350,852	2,644 ← **diameter 11**
12	782,536	0
13	90,280	0
14	276 ← diameter 14	0
	3,674,160	3,674,160

For 3x3x3 Rubik's cube: ≈ 1.4 trillion states

diameter ≤ 26 but unknown

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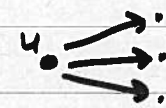
Graph representations on computer

Operations to support:

- create empty graph
- add vertex or edge
- remove vertex or edge
- list vertices adjacent to given vertex u
- test if edge $(u,v) \in E$



(neighbors)
$Adj[u]$

① **Adjacency list representation:**

V = set of vertices (hashable)

Adj: dictionary

$Adj[u]$ = list of vertices adjacent to u

note: can have multiple graphs on same set of vertices

② **Object-oriented variations:**

[vertex u is an object
[u.neighbors ($= Adj[u]$) neighbors of u

[edges can be objects too
[u.edges = list of (outgoing) edges from u

Above reps are good for sparse graphs, where $|E| \ll |V|^2$

since space requirement is $\Theta(V+E)$

don't bother with $|\cdot|$'s inside O/Θ

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③ <u>Adjacency matrix representation</u>

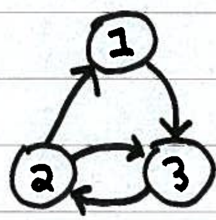
Assume $V = \{1, 2, \ldots, |V|\}$

Let $A = (a_{ij}) = |V| \times |V|$ matrix

where $a_{ij} = \begin{cases} 1 & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases}$

i = row
j = column

e.g.



$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ A = & \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \end{array}$$

good for <u>dense</u> graphs (where $|E| \approx |V|^2$)

space required $= \Theta(V^2)$

cool properties: A^2 gives length-2 paths

can test quickly if $(i,j) \in E$

④ <u>Implicit graphs</u>

Adj(u) is a <u>function</u> returning list of neighbors

no space needed! (except during search, to keep track of places visited...)

can do this for Pocket Cube

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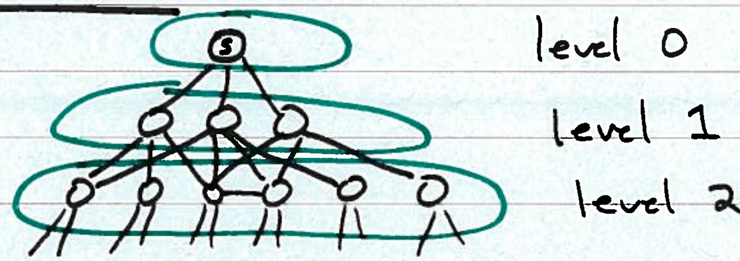
High-level overview of next two lectures:

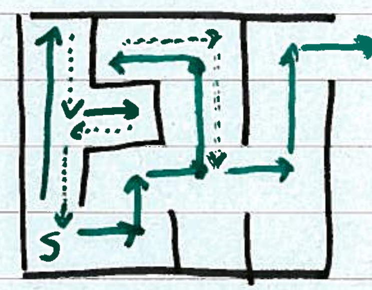
Breadth-first search:



explore "level-by-level"
mark vertices once discovered (need space $|V|$) - relevant to Pocket Cube
all unmarked vertices adjacent to level k ⇒ level $k+1$, mark them.
Finds shortest paths.

Depth-first search:

Like exploring a maze
back track when necessary
to find unexplored edges,
explore them.
mark vertices to avoid getting stuck in a loop.



Both take time $O(V+E)$