1. A hash table of size $m$ is used to store $n$ items, with $n < \frac{m}{2}$. Open addressing is used for collision resolution. Assume simple uniform hashing.

(a) Show that for $i = 1, 2, \ldots, n$, the probability that the $i$th insertion requires strictly more than $k$ probes is at most $2^{-k}$.

(b) Show that for $i = 1, 2, \ldots, n$, the probability that the $i$th insertion requires strictly more than $\lceil \log n \rceil$ probes is at most $\frac{1}{n^2}$.

(c) In this scenario, does looking up a key require more (or fewer) probes than inserting a key? Would this change if the table contained $n \leq \frac{m}{3}$ items?

2. A hash table of size $m$ is dynamically resized as items are inserted and deleted such that $m/5 < n < 4m/5$, where $n$ is the number of items in the table. Show that any sequence of $k$ INSERTs and DELETEs on this table requires $\Theta(k)$ time (that is, each INSERT or DELETE operation runs in amortized constant time). Assume simple uniform hashing. Further assume that resizing (shrinking or growing) a table of $n$ elements requires $\Theta(n)$ time. Assume that initially, $n = \frac{2m}{5}$.

If there were no initial restriction on $n$, does our bound of $\Theta(k)$ time for $k$ operations still hold? (What happens if $n$ is initially $(m + 1)/5$ and we perform only $k = 1$ deletion?) How many operations do we have to perform in order to guarantee that our $\Theta(k)$ bound is valid?