Parallelizing Spectral Elements

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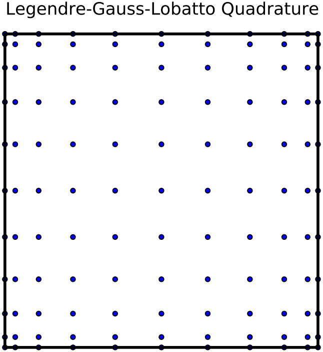
What are spectral elements?

Spectral elements are a numerical PDE method.
Variables are replaced by polynomials, and integrated by a quadrature.



Cells only communicate through surface.

What am I solving?

I'm solving the multigroup neutron diffusion equation:

$$\overbrace{\frac{1}{v_i}\frac{\partial \phi_i}{\partial t}}^{\text{change in time}} - \underbrace{\nabla \cdot D_i \nabla \phi_i}_{\text{diffusion}} = \overbrace{S(\phi_i)}^{\text{source}}$$

The problem I am solving is known as the LRA benchmark.

It is a symmetric 2D nuclear reactor with group-collapsed cross sections.

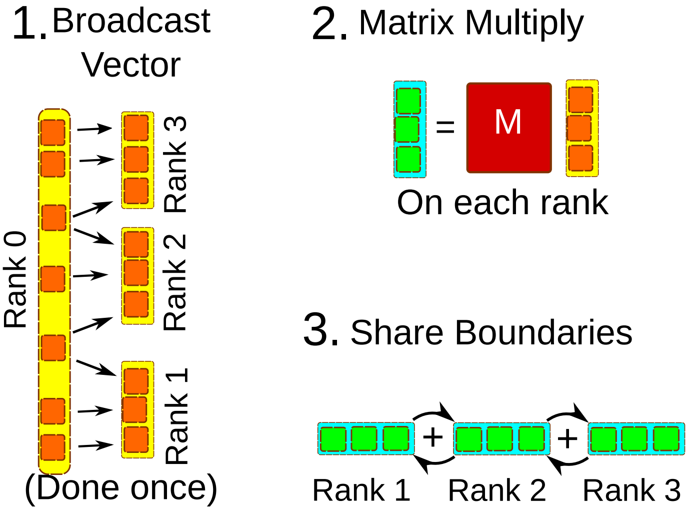
How did I make it parallel?

To advance in time implicitly, I implemented GMRES. GMRES requires 3 components:

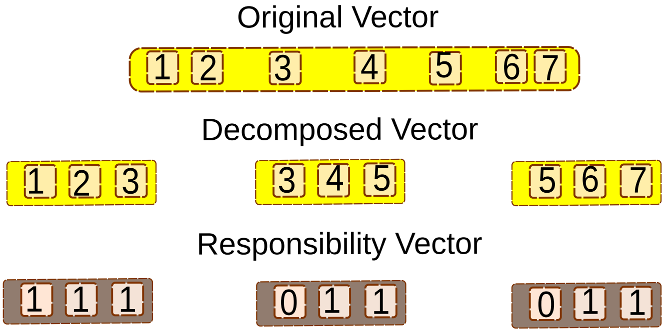
- A parallel `matmul`
- A parallel `norm`
- A parallel orthogonalization

(Also, GMRES + power iteration can solve the generalized eigenvalue problem for steady state)

matmul


1. Broadcast Vector
Rank 0
Rank 3
Rank 2
Rank 1
(Done once)
2. Matrix Multiply
=
M
On each rank
3. Share Boundaries
+
+
Rank 1
Rank 2
Rank 3

norm



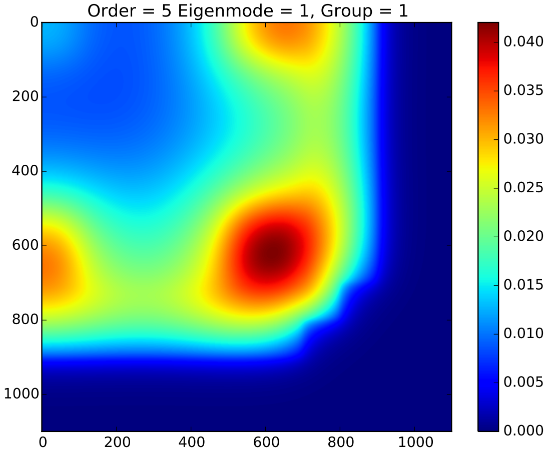

```
v[1] = sumabs2( [1 2 3] .* [1 1 1] )

norm = sqrt(sum(v))
```

Orthogonalization

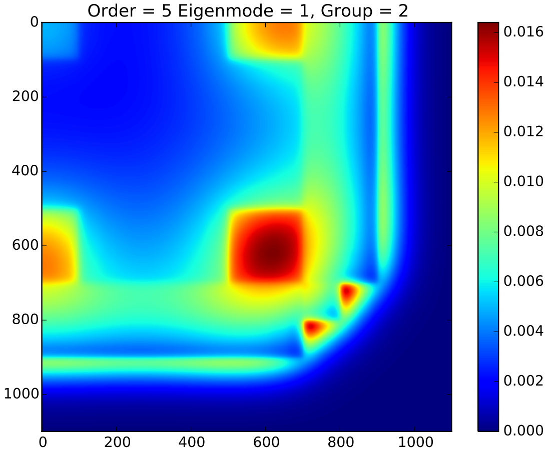
Orthogonalization is simple:

1. Perform dot product like the norm
2. Broadcast it to all cores
3. Do Modified Gram Schmidt with decomposed vectors

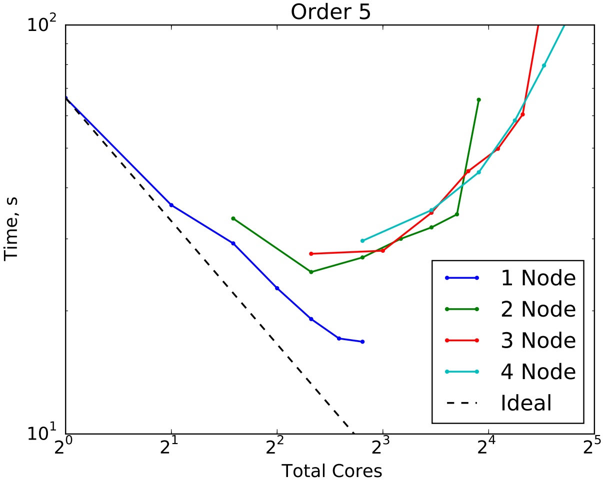
Steady State Results – Group 1



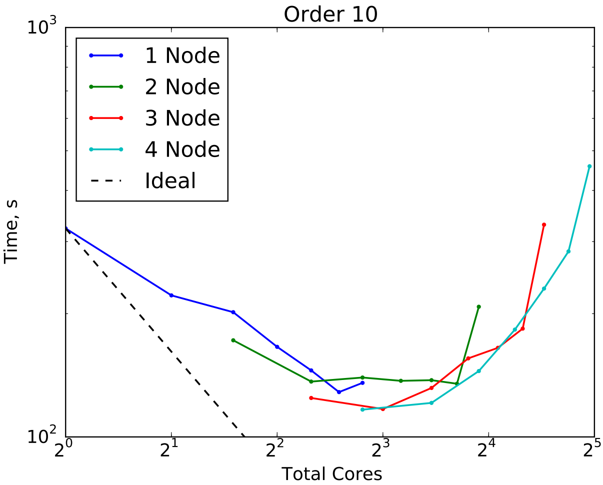
Steady State Results – Group 2



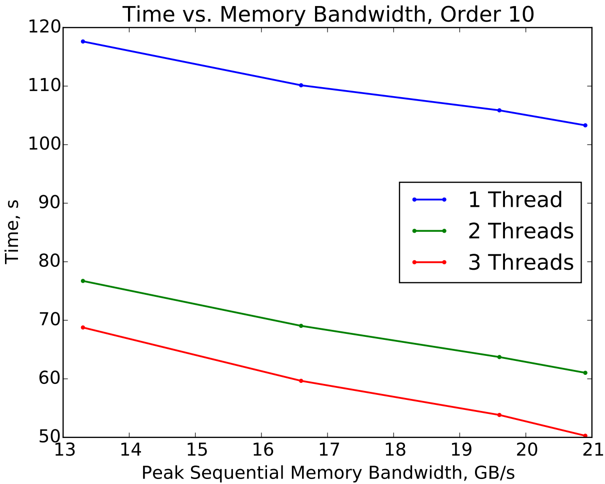
Time Used - Order 5



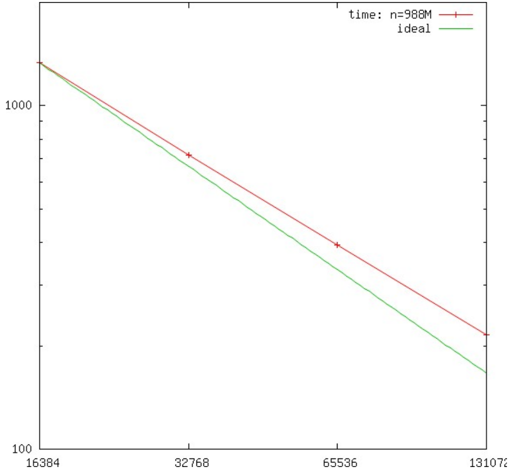
Time Used - Order 10



Time vs. Bandwidth



Spectral elements can be done far better than this though:



(from http://www.mcs.anl.gov/ fischer/sem1b/)